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Modeling and projecting Nepal's Mortality and Fertility

Chapter 1

Introduction of Mortality

1.1 Introduction and overview

The information on a country's mortality plays a dominant role in national planning [20]. The mortality data indicate country's current and future demographic situation. Moreover, they help identify the socio-economic and health progress. They chart progress in areas of increasing the expectation of life and avoidance of premature deaths, which are of utmost human concern. They are sensitive indicators of health and development programs. Thus they can be very helpful in the evaluation of the success of programs already launched. Finally, mortality levels are important because of their relationship with labor productivity and fertility.

The study of the mortality conditions in developing countries is of utmost importance. Because, the largest percentage of the population and an even larger percentage of death are found there. There has been recent improvement in the availability and the quality of the mortality data. Nevertheless, there are very few developing countries where the most reliable information on mortality, resulting from a complete death registration and combined with population census is available. The information on mortality particularly in countries with limited and defective data are also available from other sources: multi round surveys concerning small sample areas; incomplete vital registration corrected for estimated under-registration; single-round retrospective inquiries in censuses and surveys about household death in some fixed time period; comparisons of age distributions in two successive censuses; retrospective questions on censuses or surveys regarding the survival of children. These sources do not provide as reliable information on mortality levels as that based upon complete vital registration.

MEASURES OF MORTALITY :

There are various measures of mortality. Some pertain to the experience of a particular age span and the others to that of the whole life.

- e_x^0 : It is the expectation of life at age x. It is a refined measure not influenced by a population's age distribution.
- CDR: (Crude Death Rate) It is defined as deaths for all ages combined divided by total mid year population. It refers to the mortality experience of the whole life. This crude indicator of mortality is highly influenced by the age composition of the population.
- ASDR: (Age Specific Death Rate ${}_nM_x$) It is identical to crude death rate, except that the deaths in the numerator and the mid year population in the denominator pertain only to a certain defined age span. The notation for the age specific death rates in a life table population is ${}_nm_x$. ASDR refers to the mortality experience of a particular age span. Here x is the age and n is the duration.
- ${}_nq_x$: It can be obtained from age specific death rate by simple arithmetic operation. It is defined as the probability of death prior to the end of the interval for someone who survives to it's beginning. The complement of the probability of death is the probability of surviving. Here x is the age and n is the duration.
- IMR: (Infant Mortality Rate) The denominator of IMR is the annual number of births, while the numerator is the number of infant deaths in that year.

Various measures of mortality are also explained in detail in [4] (p. 90). While studying mortality trends, it is hazardous to rely upon these measures. It is even risky to use them to make inferences about a change in trend. One advantage of the survey data on mortality compared to civil registration data is the former provide a richer detail on difference in mortality by socio-economic and other characteristics.

Availability of mortality data in Asia.

The availability of adequate data for mortality analysis in Asia falls somewhere between that of Africa and Latin America. Death registration is incomplete or non existent in most parts of Asia. Around 1950, only three countries and territories (Hongkong, Singapore and Peninsular Malaysia) claimed to have complete vital registration, where according to the definition, 90 percent or more of all deaths were recorded. By 1975 only one country, Sri Lanka, was added to the list. The four areas are arguably atypical and together account for less than 2 percent of Asia's population. For most of Asia including India and China, the level of mortality must be estimated frequently by means of indirect techniques applied to data derived from the sample surveys.

Age and Sex pattern of mortality:

Mortality rates are usually very high at birth, but decline rapidly to a minimum at the early teens, and then increase continuously, at a varying pace, towards the end of the life span. This results in the typical U-shaped curve of mortality associated with low life expectation.

In the developed world or in countries with high life expectation, the decline of mortality level with deaths are increasingly postponed until the older ages, and the curve gradually becomes J-shaped. Further, the death rates of males in these countries at every age were higher than that of females. In Africa and Latin America, on the other hand, female mortality was sometimes higher than of male mortality in early childhood and reproductive span. The countries in Asia exhibit higher female than male death rates to a greater extent than the developed world. Early births through child bearing age groups are particularly affected. In some countries, high female mortality is found at advanced ages, where many women, often widows, are left without the support of their families.

1.1.1 General overview of mortality laws

The demographic analysis of mortality is dependent on vital statistics, describing the event of death and the characteristics of the decedent, but the presence of limited and defective data is one of the big obstacles lying in the path of mortality research.

Crude and age-specific death rates, cause specific rates and ratios and adjusted death rates of many varieties are just a few of the many indices used in mortality research [34]. Mortality level of a country depends on its health care facilities which in turn is dependent upon the country's economic and financial development.

In Nepal the vital registration system is missing, and the data on deaths, obtained through censuses and surveys, are also poor in quantity and quality. With census data suffering from serious problems of coverage and quality, periodic surveys have become the main sources of mortality data of Nepal. The demographic sample surveys 1974-75, 1976, 1977-78, and 1986-87 have collected information on deaths by age and sex. The age specific death rates for males and females are adjusted for under-reporting [11]. The age specific death rate for year 1991 is obtained from male and female life tables Nepal 1991.

Like Nepal, other developing countries also have such problem of poor data. Records of mortality in African communities are even more scanty and unreliable than those of fertility. Deaths are registered in only a few countries and regions, and in most of these very incompletely [7]. For example, a principal source of distortion in the infant mortality data in Nepal is the omissions by older women of births of children who died in infancy many years ago, which helps explain the decline in infant mortality, with increasing age of mother [57]. In Indian context, as stated by Biswas [4], (p. 337), general apathy of the public and lack of permanent government organisation whose whole responsibility is the registration of vital events are some of the many reasons.

There are three approaches, in which the age pattern of mortality have been formulated [54].

The three different approaches are described briefly below.

1. Functional Form

   Mortality laws can be represented in terms of functional form of mathematical expression with several parameters.

   A.deMoivre proposed the function $\mu(x) = \frac{1}{w-x}$, where w represents the highest attainable age and $\mu(x)$ is the force of mortality. It appeared first in the book *Annuities on lives*(1725). This model, as deMoivre himself noticed, gives relatively good results at older ages, but is inadequate for the younger [31].

   Gompertz in 1825 proposed a function $\mu(x) = BC^x$ and Makeham improved and proposed it in 1860 as $\mu(x) = A + BC^x$. Death can occur because of one of the several causes and these causes (or many of them) act independently, then the Gompertz formula is one of the limiting distributions that result. In 1960, Brillinger formulated a general form of Gompertz curve, with the force of mortality.

   $$\mu(x) = \sum_i H_i(x - B_i)^{C_i - 1} + \frac{A_i}{(b_i - x)^{C_i + 1}} + E_i {d_i}^x$$

   The constant term of Makeham's law is obtained, when one of the d_i is set to 1.

   Another functional expression by Perks:

   $$\mu(x) = \frac{A + BC^x}{kc^{-x} + 1 + Dc^x}$$

British actuaries have used the following law.

$$\frac{q_x}{p_x} = A - Hx + BC^x$$

so,

$$p_x = \frac{1}{A - Hx + BC^x}$$

where p_x is the probability of survival $p_x = 1 - q_x$

Heligman and Pollard proposed:

$$\frac{q_x}{p_x} = A^{(x+B)^C} + De^{-E(lnx - lnF)^2} + \frac{GH^x}{1 + GH^x}$$

This is an eight parameter model, which has been quite successful in modeling mortality of countries with accurate and sufficient data. But in a country like Nepal, with a very limited and defective data, the estimation of such a large number of parameters was very cumbersome. This method has not produced promising results. Efforts have been made not only at modeling $\mu(x)$, but other life table columns, too.

The model ranges from simple linear models with a small number of parameters to complicated non-linear models with large number of parameters. Here works [44], [41], [42] on modeling life table functions and mortality functions are of interest.

In countries with a long history of recorded data, time series analysis of parameters can be done [39], [56], [38], [10], [47], [33].

Time Series models require long series of data of at least several decades, [19], page 215-221. But unprecedented and abrupt changes in contraceptive technology and practice make, this assumption hard to accept. An additional difficulty with the time-series model is that assumption about trends are strictly mechanical 2and fail to incorporate our best prior information.

In Stochastic demographic forecasting, statistical time series methods are combined with demographic models to produce plausible long run forecasts of vital rates with probability distribution [35], [58]. Brillinger [8] has conceptualized and formalized stochastically the natural variability of vital statistics. This would support their use in comparisons and other analysis.

2. Numerical Tables

   These tables are generated from the statistical summary of large vital statistics data.

   Model life tables have been formulated to represent the entire facts of a life table in one or two dimensional space. A point in one or two dimensional space, is identical to ordinary, real life tables, except that they relate to no particular place or time [46]. Different sets of life tables have been published since 1950s. Some of the model life tables are discussed below.

   (a) The 1955 United Nations set

       First in 1955, and then revised in 1956, they are rarely used in practice now. Their simplicity makes them mentionable here. For each sex, 158 empirical life tables were the basic ingredients, which were subjected to complex regression analysis, in order to produce a set of 24 model life tables for each sex. These 24 model life tables were classified according to the mortality levels e_0 which varied from $e_0 = 20$ (level 0) to $e_0 = 73.9$ (level 115). These model life tables are criticized as being inflexible, as once the mortality level has been chosen, the entire corresponding life table has to be selected. The entire life table based on only one parameter e_0 is represented by one point (e_0), on one dimensional space.

   (b) Coale and Demeny model life tables

       These are one of the widely used model life tables. First edition was published in 1966 and followed by their the second edition was in 1983. Similar to United Nations model life table, they comprise of 25 levels for each sex, though these are called levels 1 to 25 rather than 0, 5, ...,115. Coale and Demeny model life tables are more flexible than U.N model life tables, and are divided into four families north, south, east and west. Each family has specific age pattern of mortality and high or low infant or adult mortality rate. The entire life table can be represented by points lying in two-dimensional space, one representing the level and another representing the family. The important characteristics of the four families are shortly highlighted below.

       i. North North family is featured by relatively low infant and old age mortalities but high adult mortality, and is based on nine life tables from Norway, Sweden and Iceland.

ii. South It is characterized by high mortality under age 5, particularly among infants, low adult mortality and high mortality over age 65. 23 Mediterranean life tables from Portugal, Spain, Sicily and Southern Italy produced South family.

iii. East
High infant and high old age mortalities relative to childhood and adult rates are the typical features of east family. It is derived from 36 Central European life tables from Austria, Germany, Bavaria, Prussia, north Italy, Poland, Czechoslovakia.

iv. West It is based on life tables from, among others, the Netherlands, Finland, France, England, Wales, Japan, Ireland, Israel, Australia, Canada and South Africa. When none of the above mentioned patterns of mortality are observed, then the West family of model life tables is recommended[12].

(c) The 1982 United Nations Set One of the main drawbacks of Coale and Demeny model life tables, derived from European life tables, is that they donot suffice for the study of mortality pattern of developing countries, but the 1982 United Nations Set overcomes this drawback. Here the accurate 72 life tables (36 male and 36 female) derived from four major groups (Latin American, Chilean, South Asian, and Far Eastern) of developing countries have been collected. A fifth General pattern was also produced which is an average of all the original empirical life tables. The main features of these five groups are:

i. Latin American This group is featured by high infant, child and adult mortalities, but relatively low old-age mortality.

ii. Chilean
This group is characterized by very high infant mortality.

iii. South Asian High mortality under 15 and over 55, but relatively low mortality at adult ages are the typical features of this family.

iv. Far Eastern This group is featured by high male death rates at older ages relative to their death rates at younger ages and very high sex ratio of mortality at the older ages.

v. General It comprises of model life tables which donot exhibit any special pattern. General patterns in the 1982 United Nations Set are similar to the west family in Coale-Demeny model life tables.

As in Coale-Demeny model life tables, these tables also have five families of life tables (for each sex) and are tabulated against expectation of life at birth from 35-75 years.

3. Relational Procedures An association is established between the observed mortality pattern and standard mortality schedule.

   (a) Brass two parameters model life tables system Brass' two parameters model life tables system is based on the assumption that different mortality schedules are linearly related to each other through logits of their respective survivorship values. Let $l_a(x)$ and $l_b(x)$ be the observed survivorship values at age x=1,.....w for the two populations a and b, then it is possible to find constants α and β such that

   $$logit[l_a(x)] = \alpha + \beta.logit[l_b(x)] \tag{1.1}$$

   or, $Y_a(x) = \alpha + \beta.Y_b(x)$ where w is the maximum possible age. So, here unlike United Nations model life tables, these are directly based on linear regression whereas the former are derived from empirical life tables.

   $$logit(1 - l(x)) = .5[ln((1 - l(x))/l(x))]$$

   If l_x lies between 0 and 1 then $logit(1 - l(x))$ lies between -∞ and ∞. Here α and β have their special meanings. α is the level of mortality and β is the ratio between the childhood and adult mortality.

   When $\alpha = 0$ and $\beta = 1$, then from (1.1) we have the same life tables of the populations a and b. Instead of population b, population a can be compared to a standard population. Any standard population with a reasonable average pattern of mortality can be chosen. Two standards are in common use. They are known as General standard [2] and African standard (Page 133)[12]. When populations of dissimilar mortality patterns are compared, S shaped curve is obtained instead of getting a straight line.

   The advantages of Brass two parameter system are the following.

      i. In a country with limited or defective data and thus, with limited points in the life table, the entire life table can be reconstructed with the help of this system. Once α and β have been calculated, the missing observations can be found out.

ii. This system doesn't require big tables or complicated computer programs.

(b) Four parameters life table system
Because of some limitations of two parameters system, it was extended to four parameter system [2].

$$l_N(x) = l_s(x) + \Psi k(x) + \chi t(x)$$

$$logit(l(x)) = \alpha + \beta.logit(l_N(x))$$

where $l_s(x)$ are the survivorship values for age x of the general standard. k(x) and t(x) are the schedules of deviations from the general standard. $l_N(x)$ may be chosen by specifying the two constants Ψ and χ. So, here with Ψ and χ an effort is made to alter the tail of the curve (infant mortality and old age mortality).

As compared to Brass two parameters α and β, there are now four parameters α, β, Ψ, χ. The main objective behind the extension from two parameters life tables system to four parameters life tables system is the inability of the former to fit in the population with unusually high or low infant or old age mortalities. That is the life tables, which are somewhat different from the normal pattern.

A somewhat different approach of not relying on several standard age patterns of different kind is mentioned in [24].

Principal components analysis of the logit of the survivorship function[45], analysis of the Brass's parameters [43], and the discussions on the problem of choosing model life tables for African countries[23], are some of the works done applying Brass's two parameters system. Zaba and Paes [3] have proposed a new method of estimating α and β parameters of the logit relational models or Brass's models. Carrier and Thuan - Jig [9] have tested Brass's model by comparing an actual and a fitted life table for a developing country.

1.1.2 Objective

Taking the background of Nepalese data into consideration, an effort is made to forecast and correct Nepal's vital rates, especially mortality and fertility. The presence of qualitatively and quantitatively poor data is taken as a challenge rather than as an obstacle.

An effort is made towards the construction of a parsimonious model, specially suited to Nepal. While achieving this goal, steps have also been taken in the direction of identification and correction of errors in the data. Missing data have been taken care of through statistical procedures. A great effort has been put on the development of models with minimum number of parameters. There are some very good models, which give not only very good results but also have large number of parameters. These models have been very successful in predicting the mortality and fertility of countries, with an abundance of accurate data.

A comparison between various countries has been made. For example Germany with accurate and abundant data has been compared with India and Nepal, representing countries in South-Asia.

1.1.3 Numerical methods

In an effort to find a suitable model, an extensive use of numerous computer software packages has been made. For extensive numerical interpolation and extrapolation, EXCEL worksheets have been very useful. In addition to that, EXCEL has been used very intensively for data mining and therefore in the correction of faulty data. A rigorous use of EXCEL in the production of tables as well as in the construction of graphs has been made. MATHEMATICA, a computer algebra system, has been used to find the fast solution of symbolic mathematical problems (such as pseudo-inverse, higher order determinants, initial solution of non-linear equations). Excel was not programmed for symbolic Mathematics. With Mathematica quick and efficient solutions of such problems were achieved.

NAG FORTRAN library system and SPSS advanced statistical system has been implemented in linear and non-linear optimization problems and modeling. In some cases they have complemented each other.

1.1.4 Data Analysis

The data had to be analysed very intensively and that which seemed useful chosen for further study. In order to proceed with modeling, the data available in a certain form were further transformed into the required form.

Age specific death rate [11] given here in Table 1.1 has been used as a data source. The abridged life table of Nepal, for example, was available for the year 1991, but for the previous years, it was converted from ASDR data using Reed and Merrell method(Table 1.1)[53].

The Reed and Merrell method.

The Reed and Merrell tables can be used to convert ${}_nm_x$ to ${}_nq_x$. The construction of these tables are based on the fact that, there is a high association between age specific death rates over an interval and the probability

Table 1.1: Age Specific Death Rates, Nepal 1971 - 1991

Age	**1991**		**1986-87**	
	male	female	male	female
0	0.101	0.111	0.110	0.128
1-4	0.010	0.020	0.023	0.033
5-14	0.007	0.003	0.005	0.005
15-24	0.002	0.004	0.004	0.006
25-34	0.004	0.006	0.006	0.007
35-44	0.006	0.007	0.012	0.008
45-54	0.010	0.045	0.017	0.045
55-64	0.018	0.034	0.022	0.045
65+	0.086	0.082	0.033	0.1345

Age	**1977-78**		**1976**		**1974-75**	
	male	female	male	female	male	female
0	0.141	0.108	0.105	0.098	0.138	0.123
1-4	0.033	0.015	0.036	0.022	0.037	0.036
5-14	0.0048	0.003	0.007	0.0052	0.0061	0.0056
15-24	0.005	0.004	0.0041	0.0043	0.006	0.0079
25-34	0.0047	0.0061	0.0042	0.0065	0.0107	0.0077
35-44	0.0067	0.0078	0.0068	0.0102	0.0148	0.0126
45-54	.021	0.011	0.021	0.025	0.035	0.025
55-64	0.036	0.023	0.029	0.039	0.048	0.038
65+	0.098	0.081	0.099	0.0162	0.108	0.1204

of dying within that interval. When ${}_nm_x$ is the age specific death rate in a stationary population, ${}_nq_x$ is the probability of death. l_x is the probability of survival from birth to age x. It can be seen from Tabel 1.2

$$ {}_nm_x = \frac{l_x - l_{(x+n)}}{\int_x^{x+n} l_t dt} $$

or,

$$ {}_nq_x = \frac{l_x - l_{(x+n)}}{l_x} $$

Assuming that over the interval n

$$ l_x = a + bx \tag{1.2} $$

then ,

$$ {}_nm_x = \frac{(a + bx - a - b(x+n))}{\int_x^{x+n} (a + bt) dt} $$

$$ {}_nm_x = \frac{-b}{a + bx + bn/2} \tag{1.3} $$

$$ {}_nq_x = \frac{-bn}{a + bx} \tag{1.4} $$

So by eleminating a+bx between equation (1.3) and (1.4)

$$ {}_nq_x = \frac{2n.{}_nm_x}{2 +_n m_x} \tag{1.5} $$

So assuming a linear relationship between l_x and x (1.2), the ${}_nq_x$ values can be estimated from equation (1.5).

Or, as an alternative l_x is assumed to be an exponential curve over an interval n, that is

$$ l_x = e^{a+bx} $$

then ,

$$ {}_nm_x = \frac{l_x - l_{(x+n)}}{\int_x^{x+n} l_t dt} $$

$$ = \frac{e^{a+bx} - e^{a+b(x+n)}}{e^{a+bx}} = 1 - e^{bn} $$

so,

$$ {}_nq_x = 1 - e^{-n.{}_nm_x} $$

For Nepal's case, the ${}_nq_x$ values are read off from the conversion tables, associating ${}_nq_x$ values to ${}_nm_x$ values. The standard table for ${}_3m_2$, ${}_5m_2$, and

Table 1.2: Abridged Life Table Males Nepal 1976-1986

Age-group	**A.S.D.R** male;1986	${}_nq_x$ Reed-Merrel Method	${}_nl_x$	${}_nd_x$
0	0.111	0.094	100,000	9449.44
1-4	0.020	0.073	90,551	6583.171
5-15	0.003	0.034	83,967	2813.630
15-25	0.004	0.042	81,154	3426.482
25-35	0.006	0.055	77,727	4250.796
35-45	0.007	0.068	73,476	4994.343
45-55	0.0451	0.374	68,482	25583.405
55-65	0.0342	0.296	42,899	12709.282
65+	0.082	0.583	30,189	17603.584
Age-group	**A.S.D.R** male;1976	${}_nq_x$ Reed-Merrel Method	${}_nl_x$	${}_nd_x$
0	0.128	0.107	100,000	10720.5
1-4	0.033	0.112	89,280	10018.749
5-15	0.005	0.005	79,261	390.962
15-25	0.006	0.006	78,870	448.532
25-35	0.007	0.071	78,421	5551.221
35-45	0.008	0.077	72,870	5636.934
45-55	0.045	0.374	67,233	25116.792
55-65	0.045	0.296	42,116	12477.479
65+	0.135	0.775	29,639	22957.082

${}_{10}m_2$ have been prepared on the assumption that the following exponential equation holds.

$$ {}_nq_x = 1 - e^{-n_n m_x - an^3 {}_n m_x{}^3} $$

Where,

n is the size of the age interval;

When ${}_nm_x$ is based on the life table values, it is called the central death rate, otherwise it is called age specific death rate;

a is a constant. These tables are read off from [28](p. 513).

Chapter 2

Some Mortality Models

2.1 Modeling l_x; The proportion of people living till age x, for Nepal

Various attempts to model several life table functions have been made. Probably the most often applied statistical method of modeling Life table functions is the logit model of Brass [5]. He found a high correlation between logits of the survivorship functions and suggested the linear equation

$$\frac{1}{2}(\frac{ln(1-l_0(x))}{l_0(x)}) = p + (\frac{q}{2})(\frac{ln(1-l_s(x))}{l_s(x)}) \tag{2.1}$$

where $l_0(x)$ and $l_s(x)$ are survivorship functions of two life tables. $l_s(x)$ reflects a standard life table. Thus the life tables can be generated by the use of a standard life table and appropriate values of the parameters p and q. Further, the standard life table on the right hand side can be given a functional form.

Here two different models are considered. The first model discussed has been has been proposed by C. Denny[17]. The second model has been given by Lee and Carter [36]. The third model given in the succeeding chapter has been given by me. The first model is very simple. It is based on very few parameters on one hand and gives very accurate results on the other hand. In the second model, the age and time effects have been studied separately.

2.2 The Model

The Model:

$$l_x = \frac{1}{1 + a(\frac{x}{105-x})^3 + b(e^{(\frac{x}{105-x})} - 1)^{(1/2)} + c(1 - e^{(-2x)})} \tag{2.2}$$

A number of criteria have led to the construction of this model. Brass logit model is the basic foundation of this model. Mitra [43] extended Brass's logit model to the model with survivorship functions of two standard life tables. He gave the standard life tables functional form as $f_1(x) = (\frac{x}{105-x})^3$, $f_2(x) = (e^{(\frac{x}{105-x})} - 1)^{(1/2)}$ and $f_3(x) = 1 - e^{(-2x)}$

$$\frac{1}{l_x} = a(\frac{1}{l_s(x)} - 1) + b(\frac{1}{l_t(x)} - 1) \tag{2.3}$$

where $l_s(x)$ and $l_t(x)$ stand for survivorship functions of two standard life tables. He further proposed [17]

$$\frac{1}{l_x} = 1 + af_1(x, m_1, n_1, \ldots) + bf_2(x, m_2, n_2, \ldots) \tag{2.4}$$

where a, b, c , ... are constants for specific life tables and $m_1, n_1, \ldots, m_2, n_2, \ldots$ etc. are constants for all life tables. Model (2.2) has two boundary conditions.

1. $\frac{1}{l(0)} = 1$
2. $\frac{1}{l(105)} = \infty$; where age 105 is the end of the life span

2.2.1 The Fitting

The model (2.2) is a non-linear model. The objective is to find non-linear least squares estimates of the parameters a, b, c. Gauss-Newton type method was the first method thought of, for solving this non-linear least squares problem. This method was later discarded in favor of the Levenberg-Marquardt method.

Before describing Newton's method and Levenberg-Marquardt method, some terms have to be explained.

The problem was to choose that value of a, b, c from the parametric space $\Re^3$, such that the $\sum_i {e_i}^2$ is minimised. As it is a non-linear problem, that value of a, b, c is chosen , which gives a minimum of $\sum_i {e_i}^2$. Where $e_i = [l(i) - \frac{1}{1+a(\frac{i}{105-i})^3+b(e^{(\frac{i}{105-i})}-1)^{(1/2)}+c(1-e^{(-2i)})}]$

Or:

$$\begin{gathered} minimise f(x) \\ x \in \Re^n \end{gathered}$$

where

$$f\colon \Re^n \to \Re$$

Here $f(x) = \sum e(x)^2$. Here X represents the values taken by the parameters a, b, c in the Parametric space $\Re^3$. The fundamental idea of a global method of unconstrained minimisation is to take steps that take us 'downhill' for the function f. A direction p from the current point x_c is chosen such that f decreases initially and then a new point x_+ in this direction of x_c is further chosen such that $f(x_+) < f(x_c)$. p is called the **descent direction** from x_c.

Line search: It is the strategy for proceeding from a solution estimate outside the convergence region of Newton's method. That is a new estimate x_{k+1}, from parametric space is selected by considering the half of a one dimensional cross section of $f(x)$, in which $f(x)$ decreases initially from x_k. An effort is made in the selection of a direction which helps retaining fast local convergence. For attaining this goal quasi-Newton direction $-{H_k}^{-1}\nabla f(x_k)$ is positive definite with $\mu_k = 0$ if $\nabla^2 f(x_k) + \mu_k.I$ is positive definite with $\mu_k = 0$ if $\nabla^2 f(x_k)$ is safely positive definite or $\mu_k \geq 0$ and $\nabla^2 f(x_k) + \mu_k.I$ is positive definite.

Model Trust region: Here the assumption that shortened steps must be in quasi-Newton direction is dropped. In short the information about f gained as the iteration proceeds is used to better estimate the length of step likely to be successful.

The non-linear least squares problem: The nonlinear least squares problem is :

$$\begin{gathered} minimise f(x) \\ x \in \Re^n \end{gathered}$$

where,

$$f(x) = \frac{1}{2}.R(x)^T.R(x) = \frac{1}{2}.\sum_{i=1}^{m} r_i(x)^2$$

where $m > n$, the residual function $R\colon \Re^n \to \Re^m$ is nonlinear in x, and $r_i(x)$ denotes the ith component of $R(x)$. First derivative of R(x) is simply the Jacobian matrix$J(x) \in \Re^{m.n}$, where $J(x)_{ij} = \frac{\partial r_i(x)}{\partial x_j}$

Also

$$f(x) = \frac{1}{2}R(x)^T.R(x)$$

$$\nabla f(x) = \sum_{i=1}^{m} r_i(x)\nabla r_i(x) = J(x)^T R(x)$$

Similarly, the second derivative

$$\nabla^2 f(x) = \sum_{i=1}^{m} r_i(x)\nabla r_i(x)^T + r_i(x)\nabla^2.r_i(x)$$

$$= J(x)^T.J(x) + S(x)$$

From Taylor's series expansion

$$f(x) = f(x_c) + \nabla f(x_c)^T.(x - x_c) + \frac{1}{2}.(x - x_c)^T \nabla^2 f(x_c)^T.(x - x_c)$$

$$= \frac{1}{2}.R(x_c)^T.R(x_c) + R(x_c)^T.J(x_c).(x - x_c) + \frac{1}{2}.(x - x_c)^T.(J(x_c)^T.J(x_c) + S(x_c)).(x - x_c)$$

From Newton's method

$$x_+ = x_c - (J(x_c)^T.J(x_c) + S(x_c))^{-1}.R(x_c)$$

The other modification of the Gauss-Newton algorithm is to choose x_+ by the trust region approach subject to

$$\underset{x_+ \in \Re^n}{min} ||R(x_c) + J(x_c).(x_+ - x_2)||_2 \tag{2.5}$$

subject to

$$||x_+ - x_c||_2 \leq \delta_c \tag{2.6}$$

The solution of (2.6) is

$$x_+ = x_c - (J(x_c)^T.J(x_c) + \mu_c.I)^{-1}.J(x_c)^T.R(x_c) \tag{2.7}$$

where $\mu_c = 0$ if $\delta_c \geq \left||(J(x_c).J(x_c))^{-1}.J(x_c)^T.R(x_c)\right||_2$ and $\mu_c > 0$ otherwise. Formula (2.7) was first suggested by Levenberg in 1944 and Marquardt in 1963 and is known as Levenberg-Marquardt method. Levenberg-Marquardt may be slowly locally convergent on large residual and non linear problems but SPSS Advanced Statistics package, has been very successful in practising Levenberg-Marquardt. Another reason for referring to Levenberg-Marquardt is that when Gauss-Newton step is too long, the Levenberg-Marquardt step is close to being in the steepest descent direction $-J(x_c)^T R(x_c)$ and is often superior to the damped Gauss-Newton step. Several versions of the Levenberg-Marquardt have been proven globally convergent [16].

Data Analysis

The first objective is to find the initial values of a, b, c. And then from the **initial set** of values, the objective is to arrive at **final set** of values of the parameters a, b, c (using Levenberg-Marquardt rule) such that $\sum_i {e_i}^2$ is minimised.

The data, l_x values for Male, Female Nepal 1991 have been obtained from *Population Monograph of Nepal* Page 129 [11]. Where as for the years 1976 and 1986, l_x values have been obtained from ASDR data [11] using Reed-Merrell method metioned on page 10. Then the a, b, c values have been estimated using equation (2.2). The value of function l_x is obtained for different values of x using equation (2.2) and is expressed in the form of three linear equations expressed in terms of three unknowns a, b, c. Then these equations are solved to get the initial values, which are further substituted in Levenberg-Marquardt algorithm to obtain the final value.

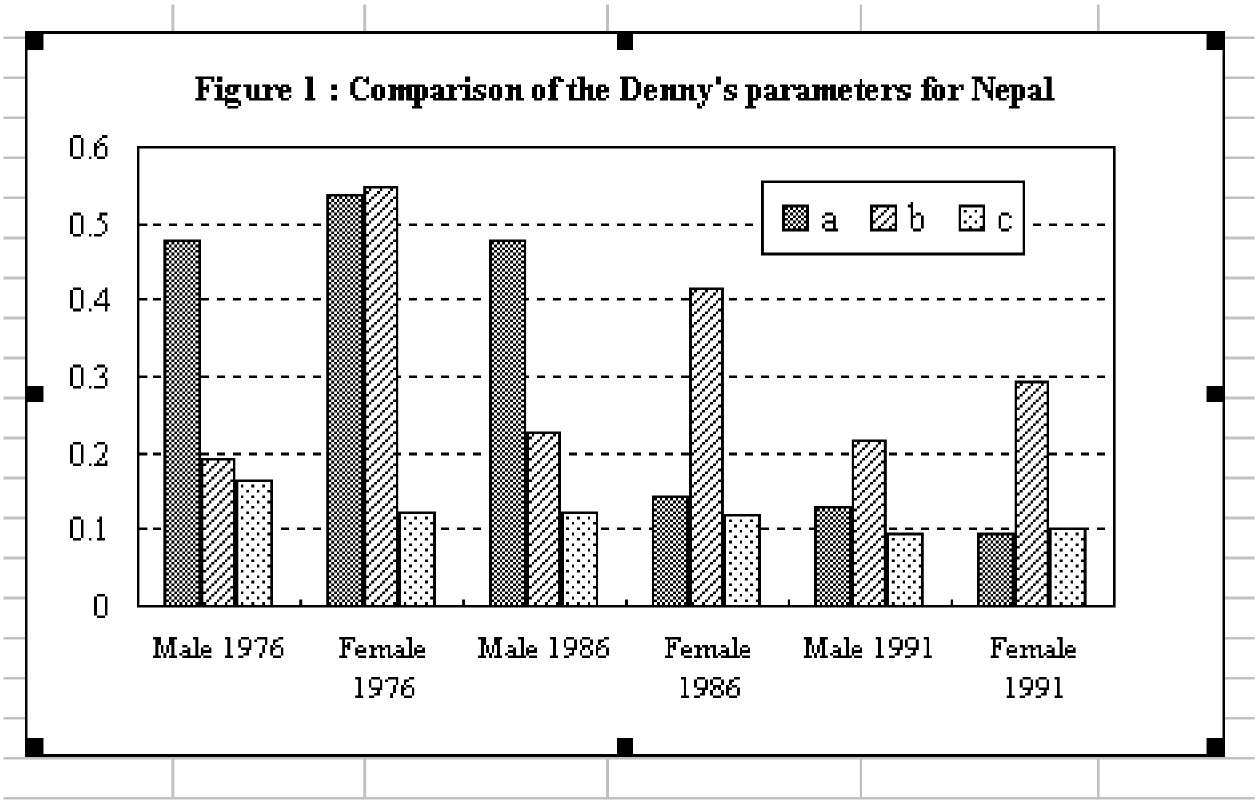
$$l_x = \frac{1}{1 + a(\frac{x}{105-x})^3 + b(e^{(\frac{x}{105-x})} - 1)^{(1/2)} + c(1 - e^{(-2x)})} \tag{2.8}$$

when $x = 0$, $l(0) = 1$
when $x = 1$, $l(1) = \frac{1}{1+a(\frac{1}{105-1})^3+b(e^{(\frac{1}{105-1})}-1)^{(1/2)}+c(1-e^{(-2)})} = .89869\ldots(i)$
when $x = 5, l(5) = \frac{1}{1+a(\frac{5}{105-5})^3+b(e^{(\frac{5}{105-5})}-1)^{(1/2)}+c(1-e^{(-10)})} = .84807\ldots(ii)$
when $x = 10, l(10) = \frac{1}{1+a(\frac{10}{105-10})^3+b(e^{(\frac{10}{105-10})}-1)^{(1/2)}+c(1-e^{(-20)})} = .83326\ldots(iii)$
when $x = 15, l(15) = \frac{1}{1+a(\frac{15}{105-15})^3+b(e^{(\frac{15}{105-15})}-1)^{(1/2)}+c(1-e^{(-30)})} = .82202\ldots(iv)$
from (i)
$.112731 = .0000009a + .098294b + .864665c\ldots(v)$
from (ii)
$.179148 = .000125a + .226431b + .999955c\ldots(vi)$
from(iii)
$.200106 = .001166a + .333171b + .999999c\ldots(vii)$
from(iv)
$.216515 = .004630a + .425864b + .999999c\ldots(viii)$

$$|A| = \begin{vmatrix} .000125 & .226431 & .999955 \\ .001166 & .333171 & .999999 \\ .004630 & .425864 & .999999 \end{vmatrix}$$

$= .000125[(.333171)(.999999)-(.999999)(.425864)]-.226431[(.001166)(.999999)-(.99999)(.004630)] + +.999955[(.001166)(.425864) - (.00463)(.333171)]$
$= -.000012 + .000784 - .001046 = -.000274$

$$a = \frac{\begin{vmatrix} .000125 & .226431 & .999955 \\ .001166 & .333171 & .999999 \\ .004630 & .425864 & .999999 \end{vmatrix}}{|A|}$$



$a = -.46715329$

$$|C| = \begin{vmatrix} .000125 & .999955 & .179148 \\ .001166 & .999999 & .200106 \\ .004630 & .999999 & .216515 \end{vmatrix} = .0000551$$

$b = \frac{|C|}{|A|} = -.081751$

$$|D| = \begin{vmatrix} .000125 & .179148 & .226431 \\ .001166 & .200106 & .333171 \\ .004630 & .216515 & .425864 \end{vmatrix}$$

$= .000125(-.013081422519) - .226431(-.000674) + +.179148(-.001046024)$
$= -.00000364$
$C = \frac{|D|}{|A|} = .00000013$

So the **initial** values for the parameters are say, $a = -.467153$, $b = -.081751$, $c = .00000013$.

Table 2.1: Parameters, Denny, a, b, c

Year/Country	**State/Sex**	**a**	**b**	**c**
1988-1992	Kerela,Male	0.08311146	0.064079253	0.012078388
India	Female	0.031647781	035551191	0.016012374
India	Gujarat,Male	0.146132741	0.102050087	0.098023979
India	Female	0.066625899	0.129378489	0.104931436
India	Bihar,male	0.103945993	0.14384458	0.095234022
India	Female	0.075884097	0.228496259	0.100184641
India	India,male	0.118174789	0.119789073	0.10410191
India	Female	.086155544	0.148796908	0.113047111
1991	Nepal,male	0.127806935	0.216718917	0.092637372
Nepal	Female	0.092668441	0.294270245	0.099753388
1986	Nepal,male	0.478461806	0.225741405	0.121034125
Nepal	Female	0.142823009	0.415499774	0.118945794
1976	Nepal,male	0.478728393	0.191606324	0.16450514
Nepal	Female	0.537632941	0.547249927	0.123682194

2.2.2 Interpretation of the parameters

It has been observed that the **final** a, b, c are always positive(Table 2.1)(Figure 1). In general, the values of the parameters a, b, c tend to decline with increasing life expectancy. This fact is verified when, the value of the parameter a, for Nepal male and female over a period of 1976-1991 is observed.

For India, the movement of the parameter a is observed for the time period 1988-92 over different regions, where the three states Bihar, Gujarat and Kerela are chosen to represent north, middle and south regions respectively, of India. There is a decline in the value of a, for the Indian female as one moves from north to south, but not for the Indian males.

The data on Nepalese female (Table 2.1) shows a decline in the value of b with increase in life expectancy. For India, b (male and female) declines from north to south, which reflects a relatively better living condition in south India than in north India. Such decline in b is not exhibited by Nepalese male data, which reflects the error in the data.

Nepal male and female data clearly reflect the trend of decrease with increase in time (life expectancy).

Denny's model has given a very good fit for Nepal as well as India (Table 2.2). Very high value of R^2 and very low value of M.S.E reflect this fact.

Table 2.2: Mean square error and R^2 of Denny's model

life tables	**Mean square error**	R^2
India male; 1988-1991	7.58E-06	0.99969
India female; 1988-1991	4.88E-05	0.99731
Nepal male 1991	1.91E-05	0.99969
Nepal female 1991	3.23E-05	0.99941
Nepal male 1986	1.01E-03	0.98482
Nepal female 1986	5.50E-04	0.98821
Nepal male 1976	1.18E-03	0.98487
Nepal female 1976	2.88E-04	0.99636

Table 2.3: Denny:Observed and Predicted l_x, Male,Female, Nepal 1976-86

Age	Male				Female			
	1986		1976		1986		1976	
	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted
0-1	1	1	1	1	1	1	1	1
1-5	0.9055	0.8874	0.8928	0.8613	0.91051	0.874363	0.88598	0.861523
5-15	0.8397	0.8531	0.7926	0.8278	0.7997	0.824375	0.77455	0.801502
15-25	0.8116	0.8201	0.7887	0.8011	0.74838	0.771276	0.7285	0.735713
25-35	0.7773	0.7859	0.7842	0.7721	0.71822	0.727293	0.68588	0.679559
35-45	0.7348	0.7339	0.7287	0.7253	0.68858	0.679599	0.61572	0.612873
45-55	0.6848	0.6404	0.6723	0.6374	0.64308	0.61792	0.53009	0.518437
55-65	0.4290	0.4814	0.4212	0.4824	0.51953	0.527076	0.37023	0.382555
65+	0.3019	0.2755	0.2964	0.2773	0.38655	0.388968	0.22467	0.220466

2.2.3 Results

Denny's model has been fitted to Nepalese and Indian data. The data of various Indian states as well as India whole have been considered. The tables of Observed and Predicted l_x (Tables2.3-2.6) show the success of the model in reflecting the survivorship scene of Nepal and India. There is a closeness between the observed and predicted values. This fact is also reflected by the Figures 2-5.

Table 2.4: Denny:Observed and Predicted l_x, Nepal, 1991

Age	female		male	
	Observed	Predicted	Observed	Predicted
0	1	1	1	1
1	0.9054	0.9079	0.8987	0.8967
5	0.8691	0.8759	0.8481	0.85734
10	0.8577	0.8584	0.8333	0.83479
15	0.8493	0.8435	0.8220	0.8160
20	0.8365	0.8292	0.8063	0.7984
25	0.8186	0.8145	0.7862	0.7808
30	0.7998	0.7982	0.7639	0.7622
35	0.7789	0.7793	0.7394	0.7417
40	0.7542	0.7563	0.712	0.7182
45	0.7236	0.7270	0.6835	0.6897
50	0.6848	0.6885	0.6505	0.6546
55	0.6338	0.6371	0.6082	0.6099
60	0.5681	0.5689	0.5549	0.5521
65	0.4832	0.4811	0.48296	0.4781
70	0.3805	0.3755	0.3930	0.3869
75	0.2647	0.2622	0.2836	0.2832
80	0.1508	0.1579	0.1697	0.1793

Figure 2 - Comparison between observed and predicted lx using Denny's model; Nepal Male 1976, 1986

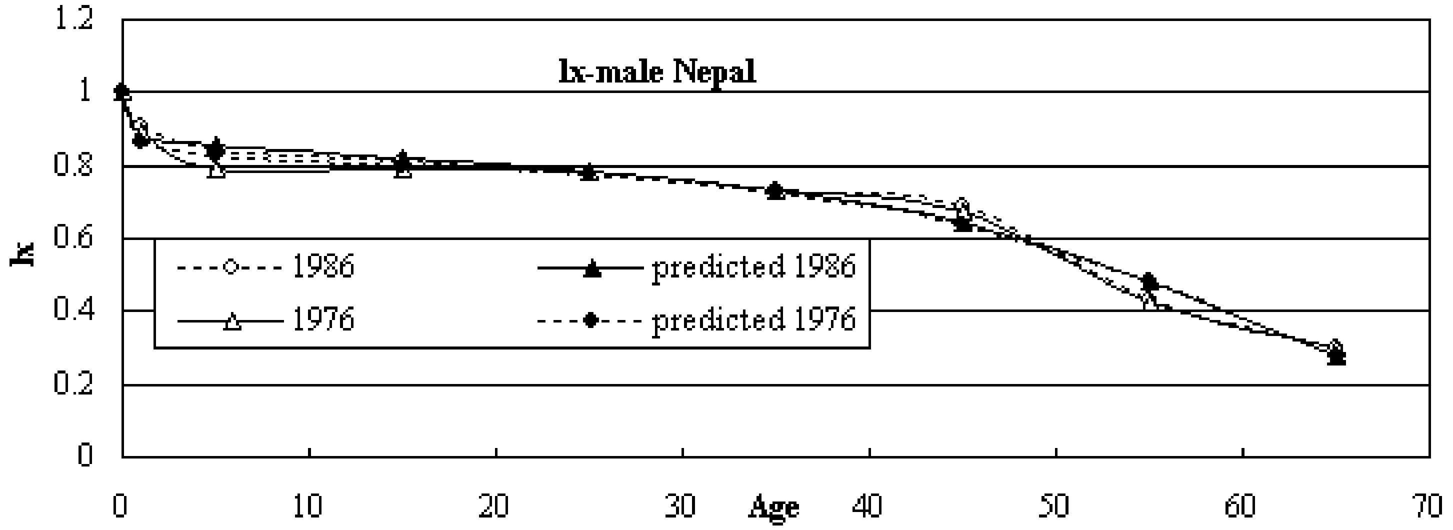


Figure 3 - Comparison between observed and predicted lx using Denny's model; Nepal Female 1976, 1986

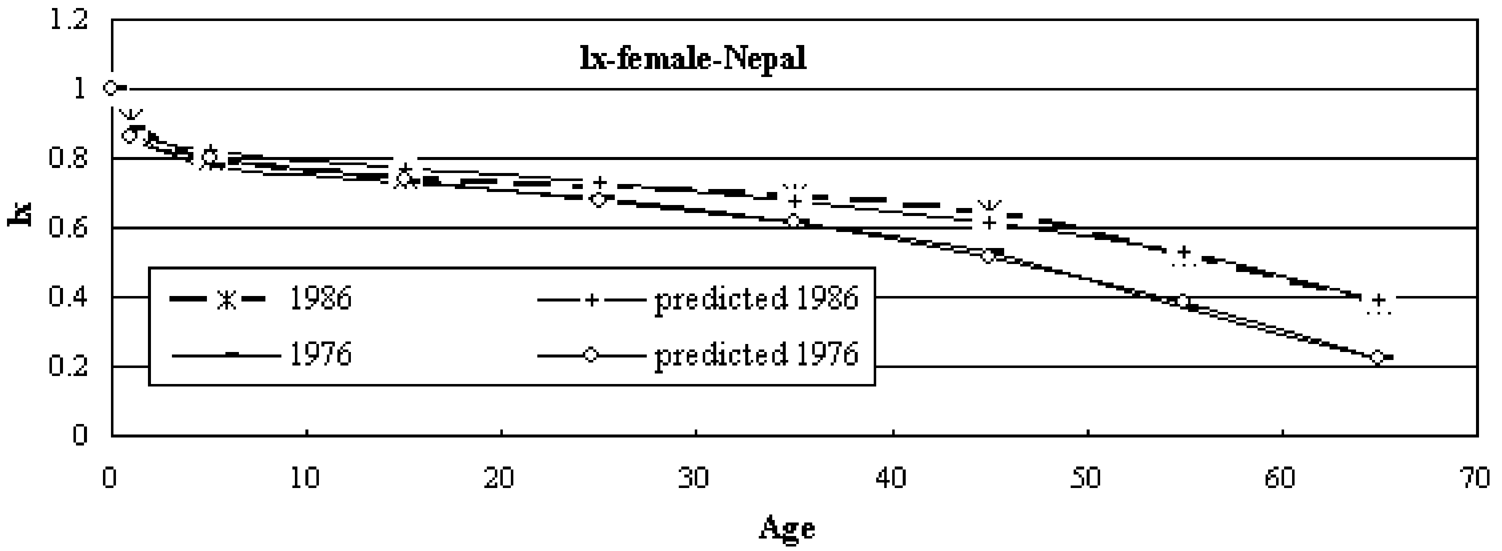


Figure 4 - Comparison between observed and predicted lx using Denny's model; Nepal Male and Female 1991

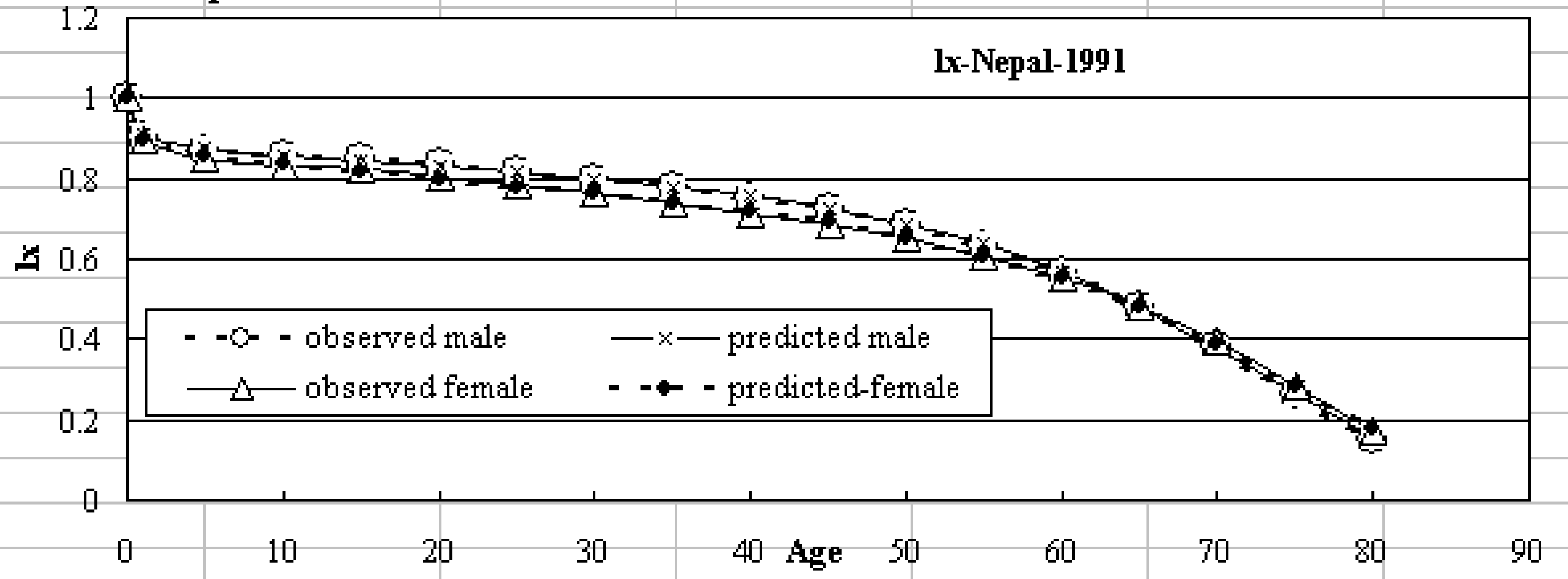


Figure 5 - Comparison between observed and predicted lx using Denny's model; India Male and Female 1988-92

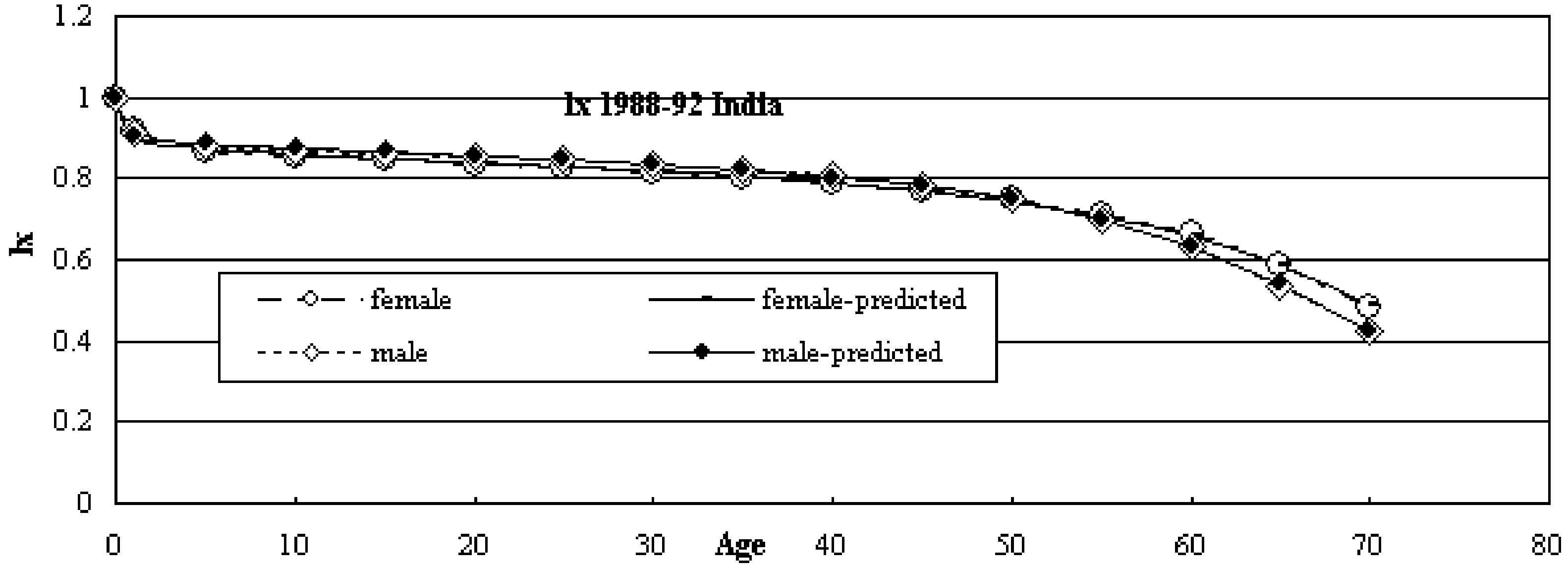


Table 2.5: Denny:Observed and Predicted l_x,Bihar and Gujarat, 1988-92

Age	Bihar				Gujarat			
	female		male		female		male	
	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted
0	1	1	1	1	1	1	1	1
1	0.9247	0.9016	0.9254	0.9120	0.9189	0.9063	0.92	0.9134
5	0.8605	0.8681	0.8833	0.8867	0.8772	0.8817	0.8895	0.8919
10	0.8406	0.8501	0.8677	0.8747	0.8660	0.8710	0.879	0.8832
15	0.8315	0.8348	0.8605	0.8643	0.8610	0.8618	0.8744	0.8755
20	0.8184	0.8204	0.8529	0.8542	0.8530	0.8529	0.8674	0.8677
25	0.8024	0.8059	0.8415	0.8435	0.8424	0.8437	0.8584	0.8589
30	0.7862	0.7903	0.8301	0.8314	0.8307	0.8334	0.8477	0.8482
35	0.7693	0.7730	0.8176	0.8169	0.8179	0.8214	0.8354	0.8344
40	0.7517	0.7527	0.8008	0.7987	0.8045	0.8066	0.8153	0.8155
45	0.7298	0.7280	0.7772	0.7745	0.7881	0.7875	0.7904	0.7889
50	0.7025	0.6967	0.743	0.741594	0.7665	0.7619	0.7502	0.7507
55	0.6636	0.6559	0.6984	0.6958	0.7307	0.7263	0.6954	0.6959
60	0.6073	0.6017	0.6308	0.6321	0.6771	0.6761	0.6225	0.619202
65	0.5290	0.5299	0.5461	0.5459	0.6069	0.6053	0.5169	0.5178
70	0.4311	0.4377	0.4352	0.4366	0.5051	0.5089	0.3946	0.3960

2.3 The Model

Here, an effort has been made to develop a parsimonious model of the pattern of change over time in these death rates, so that variation in a single parameter can generate the main outlines of the observed pattern[31]. No model is so perfect that it can reflect all age time variation. This fact has to be accepted right at the beginning[36]. The method being used here has most in common with method used by Lee and Carter 1992[36]. Let $m(x, n, t)$ be the ASDR, age specific death rate for age group $[x, x+n]$ in the year t. This matrix of death rates is fitted by the model:-

$$ln[m(x, n, t)] =_n a_x +_n b_x k_t + e_{x,n,t} \tag{2.9}$$

or,

$$m(x, n, t) = e^{_na_x +_n b_x k_t + e_{x,n,t}} \tag{2.10}$$

For appropriately chosen sets of age-specific constants, ${}_na_x$, ${}_nb_x$ and time varying index k_t, which is solely time dependent constant.

Table 2.6: Denny:Observed and Predicted l_x,Kerela, India whole, 1988-92

Age	Kerela				India Whole			
	female		male		female		male	
	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted
0	1	1	1	1	1	1	1	1
1	0.9825	0.9829	0.9776	0.9835	0.9172	0.8990	0.9150	0.9076
5	0.9753	0.9765	0.9703	0.9741	0.8681	0.8720	0.8812	0.8840
10	0.9727	0.9729	0.9668	0.9676	0.8546	0.8601	0.8701	0.8740
15	0.9706	0.9697	0.9643	0.9618	0.8478	0.8498	0.8640	0.8653
20	0.9676	0.9664	0.9600	0.9559	0.8370	0.8399	0.8565	0.8566
25	0.96399	0.9629	0.9532	0.9493	0.8255	0.8297	0.8463	0.8473
30	0.9597	0.9588	0.9449	0.9413	0.8133	0.8185	0.8355	0.8363
35	0.9540	0.953	0.9339	0.93103	0.8013	0.8055	0.8226	0.8228
40	0.9462	0.9468	0.9175	0.9170	0.7875	0.7898	0.8059	0.8051
45	0.9358	0.9373	0.8974	0.8970	0.7708	0.7698	0.7831	0.7809
50	0.9212	0.9233	0.8655	0.8677	0.7482	0.7433	0.7483	0.7470
55	0.9017	0.9021	0.8191	0.8241	0.7138	0.7070	0.6997	0.6988
60	0.8697	0.8692	0.7556	0.7598	0.6632	0.6565	0.6311	0.6311
65	0.8206	0.8173	0.6643	0.6675	0.5876	0.5862	0.5370	0.5396
70	0.7339	0.7358	0.5509	0.5438	0.4839	0.4915	0.4263	0.4251

$k[_t$ varies with time, it is an index of level of mortality.

$$\text{when } k_t = 0 \qquad m(x,n,t) = e^{n a_x}$$

$e^{n a_x}$ is the general shape of the curve independent of the time factor.

$$\frac{d}{dt}\ln[m(x,n,t)] =_n b_x \frac{d}{dt} k_t \tag{2.11}$$

From equation (2.11) it can be easily seen that

$$\text{if} \qquad k_t \to -\infty \qquad ,\text{then } m(x,n,t) \to 0$$

So, this restricts the fact that $m(x,n,t)$ is negative. ${}_n b_x$ tells, which rates change rapidly and which less rapidly. $e_{x,n,t}$ is with mean 0 and variance σ^2 and reflects age and time specific influences which cannot be captured by the model.

2.3.1 Fitting the model

The model considered is

$$ln[m(x,n,t)] =_n a_x +_n b_x k_t + e_{x,n,t} \tag{2.12}$$

For given $m(x,n,t)$ the objective is to estimate age dependent parameters ${}_na_x$ and ${}_nb_x$ and time dependent parameters, k_t. This method cannot be fitted by ordinary least squares as there are no independent regressor variables on the R.H.S of (2.12).

Further, the model (2.12) doesnot posses a unique solution of ${}_na_x$ and ${}_nb_x$ and k_t.

If ${}_na_x = a$, ${}_nb_x = b$, $k_t = k$ is the solution of (2.12) then

$$ln[m(x,n,t)] = a + bk + e$$

$$ln[m(x,n,t)] - (a + bk + e) = 0 \tag{2.13}$$

if ${}_na_x = a - bc$, ${}_nb_x = b$, $k_t = k + c$ then substituting in (2.12);

$$ln[m(x,n,t)] = a - bc + b(k + c) + e = a + bk + e \tag{2.14}$$

It can be seen that $a - bc$, b, $k + c$ also satisfy (2.12). Similarly if a, b, k is solution then a, bc, k/c also satisfy (2.12). To get a distinct solution following condition has been imposed (normalization).

$$\sum_x {}_nb_x = 1$$

$$\sum_t k_t = 0$$

$$\sum_t \ln[m(x,n,t)] = a_x \sum_t + {}_nb_x \sum_t k_t$$

$$a_x = \frac{\sum_t \ln m(x,n,t)}{\sum_t}$$

So under the conditions the sum of ${}_nb_x$ w.r.t is 1 and the sum of k_t w.r.t is 0. ${}_na_x$ is the average of $ln[m(x,n,t)]$ over time.

Data Analysis: The model is fit to Nepalese data for years 1974/75, 1976, 1977/78, 1986/87, 1991 for males and females separately. For the year 1991, the data had to be analyzed and expressed in suitable form. Only then steps towards fitting of model to the data could be taken. The ASDR 1991 has been calculated from the life table 1991(male); [11]. In order to make age groups for ASDR 1991 compatible with ASDR for years 1974/75, 1976, 1977/78, 1986/87 following method has been used.

$$M(a, a'') = \frac{\frac{M(a,a')}{K(a',a'')} + \frac{M(a',a'')}{K(a,a')}}{\frac{1}{K(a',a'')} + \frac{1}{K(a,a')}}$$

Where, M(a, a') is the age specific death rate in the age group $a - a'$.
$M(a, a'')$ is the age specific death rate in the age group $a - a''$.
$M(a, a', a'')$ is the age specific death rate in the age group $a' - a''$.
$K(a, a')$ is the total number of people in the age group $a - a'$.
$K(a', a'')$ is the total number of people living in the age group $a' - a''$.

$$M(a, a') = \frac{D(a, a')}{K(a, a')}$$

$$D(a, a') = M(a, a')K(a, a')$$

Similarly,

$$D(a', a'') = M(a', a'')K(a', a'')$$

$$M(a, a^{''}) = \frac{D(a, a'')}{K(a, a'')} = \frac{D(a, a') + D(a', a'')}{K(a, a') + K(a', a'')}$$

or,

$$M(a, a'') = \frac{D(a, a'')}{K(a, a'')} = \frac{M(a, a')K(a, a') + M(a', a'')K(a', a'')}{K(a, a') + K(a', a'')} \tag{2.15}$$

$D(a, a')$ is the total number of deaths in the age group $a - a'$.
$M(a, a'')$ is the age specific death rate of the combined age group $a - a''$

In this way, the different tables have been made compatible with respect to the age group.

2.3.2 Steps in fitting the model

From equation (2.12), $ln[m(x, n, t)] =_n a_x +_n b_x k_t + e_{x,n,t}$

$$\ln[m(x, n, t)] - {}_n a_x =_n b_x k_t + e_{x,n,t} \tag{2.16}$$

1. Logarithmic Transformation The data was available in the form m(x, t) and then it had to be subjected to the logarithmic transformation.

2. Calculation of ${}_n a_x$

$$ln[m(x, n, t)] - {}_n a_x = {}_n b_x k_t \tag{2.17}$$

$$\sum_t ln[m(x, n, t)] - \sum_t {}_n a_x = \sum_t {}_n b_x k_t$$

$$\sum_t k_t = 0$$

$$_na_x = \frac{\sum_t ln[m(x,n,t)]}{\sum_t 1}$$

$$_1a_0 = \frac{\sum_t ln[m(x,n,t)]}{5}$$

$$= \frac{.18366947 + .08864301 - .06693652 - .055177 - .15019945}{5}$$

$$= \frac{ln[m(0,1,1974/75)] + ln[m(0,1,1976)] + ln[m(0,1,1977/78)]}{5}$$

$$+ \frac{ln[m(0,1,1986/87)] + ln[m(0,1,1991)]}{5}$$

$$= -2.1412479(male)$$

Similarly the rest $_na_x$ are calculated.
$_4a_1 = -3.813069$, $_10a_5 = -5.32660713$, $_{10}a_{15} = -5.46059706$, $_{10}a_{25} = -5.19862948$, $_{10}a_{35} = -4.87526471$, $_{10}a_{45} = -3.96114437$, $_{10}a_{55} = -3.44022433$, $a_{65} = -2.284686$

3. Calculation of k_t

$$\sum_x ln[m(x,n,t)] - \sum_x {_na_x} = \sum_x {_nb_x} k_t$$

as,

$$\sum_x {_nb_x} = 1$$

$$k_t = \sum_x \ln[m(x,n,t)] - \sum_x {_na_x}$$

For the year 1974/75, it is k_1

$k_1 = \ln m(0,1,1974/75) - {_1a_0} + \ln m(1,3,1974/75) - {_3a_1} + \ln m(5,9,1974/75)$

$-{}_9a_5 + \ln m(15, 9, 1974/75) - {}_9a_15 + \ln m(25, 9, 1974/75) - {}_9a_25$

$+ \cdots + \ln m(65, 9, 1974/75) - {}_9a_65$

$$k_1 = .06549311(male)$$

$$k_2 = 2.168341$$

4. Calculation of ${}_nb_x$: The model is reduced to:

$$ln[m(x, n, t)] - {}_na_x = {}_nb_xk_t + e_{x,t}$$

Now ${}_na_x$ and the initial values of k_t are known. The next step has to be taken towards the estimation of the parameter ${}_nb_x$. As the model is not a proper regression equation, without a proper independent and dependent variable and is not a k_t full rank non singular matrix, singular value decomposition comes to the rescue.

Now with the help of singular value decomposition the value of ${}_nb_x$ is found.

As mentioned on Page 170 ,[26]

$y = bk$ is a system of linear equations, where k is a singular matrix.

$$yk^+ = bkk^+$$

$$yk^+ = b$$

where k^+ is the pseudo-inverse of k. For a general matrix A, the singular value decomposition of A.

$$A = USV^T$$

where U and V are orthogonal matrices and S is a diagonal matrix. Let A^+ be the pseudo-inverse of A

$$A^+ = (USV^T)^+ = (V^T)^+ S^+U^+ \tag{2.18}$$

because the matrices V^T and U are orthogonal, their pseudo-inverses are simply their ordinary inverses.

$$(V^T)^+ = V$$

$$U^+ = U^T$$

From (2.17), the pseudo-inverse of a rectangular diagonal matrix S has to be defined.

Let D be an $m \times n$ diagonal matrix. $D = diag(d_i)$. The pseudo-inverse of D, denoted by $D+$, is an $n \times m$ diagonal matrix $diag({d_i}^+)$ whose i-th element ${d_i}^+$ is given by

$$ {d_i}^+ = \begin{cases} 1/d_i & : \quad if d_i \neq 0 \\ 0 & : \quad if d_i = 0 \end{cases} $$

The dimensions of D^+ are the reversed dimension of D ie if D is $m \times n$ D^+ is $n \times m$. A non zero element in D corresponds to its reciprocal in D^+, but a zero diagonal element in D is associated with a zero diagonal element in D^+. The pseudo-inverse of k_t of equation (2.17) was computed using Mathematica, the Computer Algebra System. Referring to equation (2.17)

$$ ln[m(x,n,t)] -_n a_x \qquad \text{is 9x5 matrix} $$

$$ {}_nb_x \qquad \text{is 9x1 matrix} $$

$$ k_t \qquad \text{is 1x5 matrix} $$

For males:

$$ ln[m(x,n,t)] -_n a_x = $$

$$ \begin{pmatrix} .18366947 & .08864301 & -.06693652 & -.055177 & -.15019945 \\ .40786341 & .38962583 & .05804956 & -.09396664 & -.76157216 \\ -.01253223 & .06751048 & -.03358564 & -.35737272 & .33598012 \\ .16227969 & .34460125 & .03444632 & .0114568 & -.55278407 \\ -.16156328 & .27874855 & .08263367 & .0136408 & -.21345974 \\ -.13038304 & .04695098 & .44404783 & -.08658042 & -.27403535 \\ .11911373 & .33241884 & .16467971 & .07868017 & -.69489246 \\ .12152817 & .3413513 & .02897662 & .0646947 & -.55655079 \\ -.03504531 & .27849486 & .1365372 & -.21513132 & -.16485543 \end{pmatrix}_{9X5} $$

$$ {}_n\vec{b}_x = \qquad \text{to be estimated} $$

$$ k_t = \begin{pmatrix} .6549311 & 2.1683451 & .84884876 & -.639755 & -3.032369 \end{pmatrix}_{1X5} $$

k_t is a singular matrix.

$$ {k_t}^+ = \begin{pmatrix} .0423746 \\ .140294 \\ .0549212 \\ -.041393 \\ -.196197 \end{pmatrix}_{5X1} $$

With the help of Mathematica

$$_n\vec{b}_x = (ln[m(x,n,t)] -_n a_x)_{9X5}(k_t{}^+)_{5X1}$$

$$\begin{pmatrix} {}_1b_0 \\ {}_3b_1 \\ {}_9b_5 \\ {}_9b_{15} \\ {}_9b_{25} \\ {}_9b_{35} \\ {}_9b_{45} \\ {}_9b_{55} \\ b_{65+} \end{pmatrix} = \begin{pmatrix} .0482953 \\ .228441 \\ -.0440299 \\ .165094 \\ .0781143 \\ .0827983 \\ .193807 \\ .161146 \\ .086333 \end{pmatrix}$$

$$\sum_x {}_nb_x \approx 1$$

Similarly for Females;

$$k_t{}^+ = \begin{pmatrix} .0916692 \\ .13401 \\ -.0137629 \\ -.0551598 \\ -.156757 \end{pmatrix}_{5X1}$$

$$_n\vec{b}_x = (ln[m(x,n,t)] -_n a_x)_{9X5}(k_t{}^+)_{5X1}$$

$$\begin{pmatrix} {}_1b_0 \\ {}_3b_1 \\ {}_9b_5 \\ {}_9b_{15} \\ {}_9b_{25} \\ {}_9b_{35} \\ {}_9b_{45} \\ {}_9b_{55} \\ b_{65+} \end{pmatrix} = \begin{pmatrix} .047286 \\ .15242 \\ .093106 \\ .097952 \\ .116372 \\ .133456 \\ .173113 \\ .125752 \\ .0605427 \end{pmatrix}$$

$$\sum_x {}_nb_x \approx 1$$

5. Re-estimation of k_t The equation

$$ln[m(x,n,t)] =_n a_x +_n b_x k_t + e_{x,n,t}$$

has been used to re-estimate k_t. The re-estimation of k_t is necessary for the following reasons

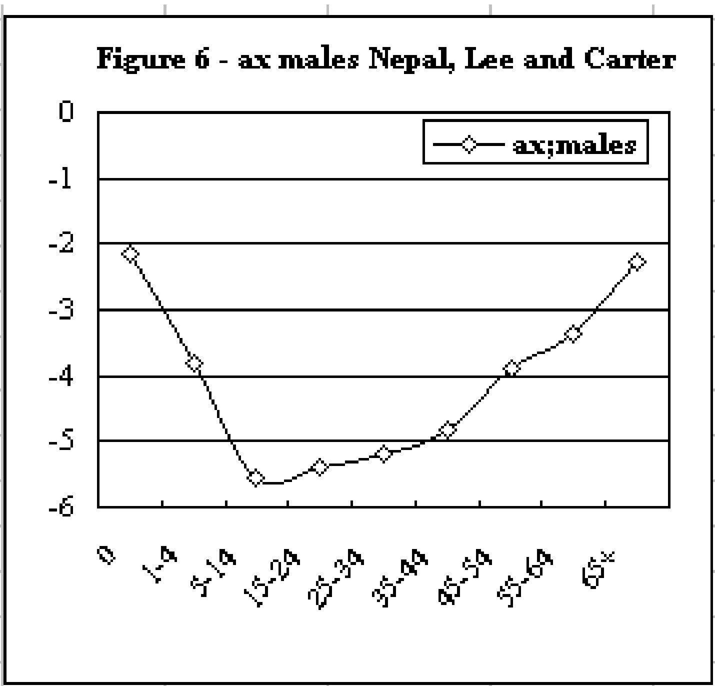


Figure 6 - ax males Nepal, Lee and Carter

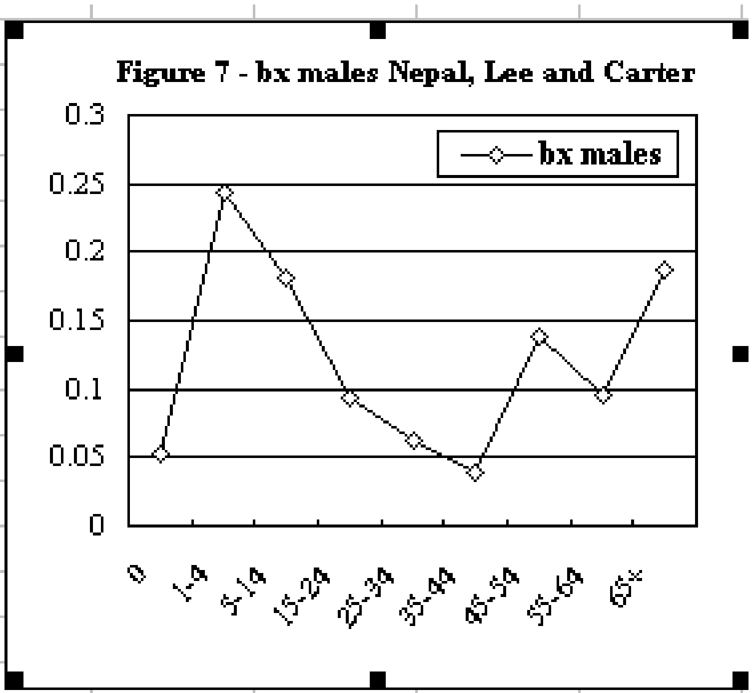


Figure 7 - bx males Nepal, Lee and Carter

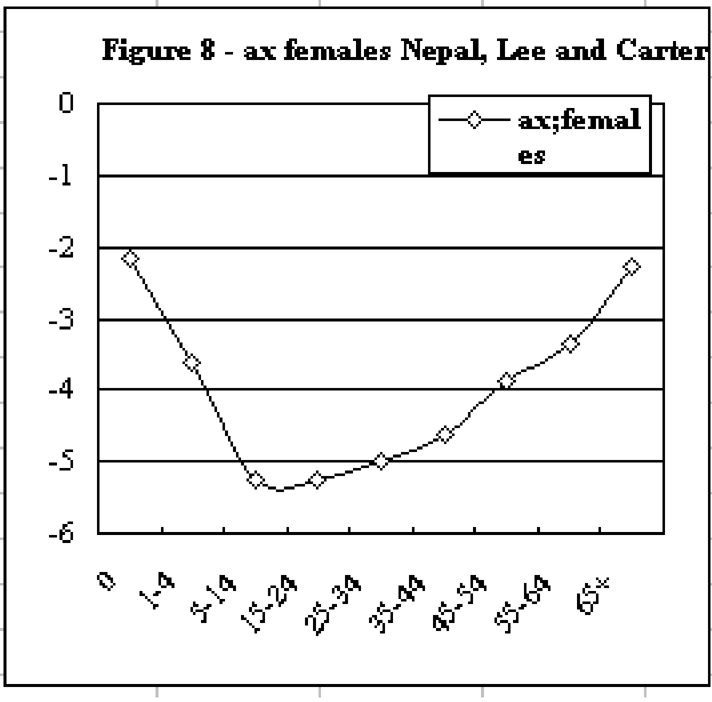


Figure 8 - ax females Nepal, Lee and Carter

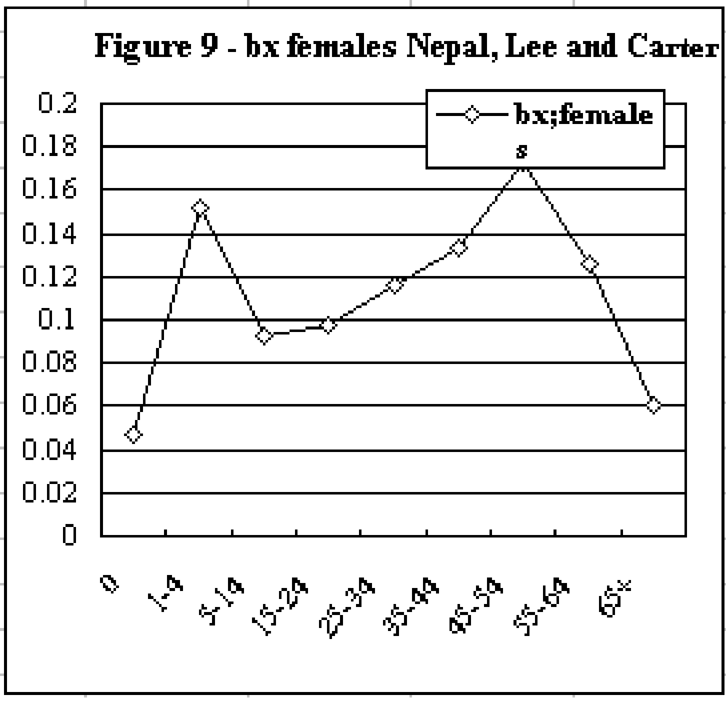


Figure 9 - bx females Nepal, Lee and Carter

(a) First stage estimation was based on $ln[m(x,n,t)]$ rather than on $m[(x,n,t)]$. Due to ln transformation, low death rates at youth receive same weights as high death rates at old age. In second stage estimation the size of age groups was considered as weights.

(b) Life tables fitted over years will be consistent with the data.

An iterative process of finding k_t which satisfies the equation 2.19 was used.

$$D_t = \sum_x e^{{}_na_x + {}_nb_x k_t} N(x,n,t) \tag{2.19}$$

where D_t is the total number of deaths in the year t,
and $N(x,n,t)$ is the size of the population in the age group $[x, x+n]$ in the year t. Here $N(x,n,t)$ is the weight allocated . $e^{{}_na_x + {}_nb_x k_t} = m(x,n,t)$ is the age specific death rate for the age group $[x, x+n]$ in year the year t. $D_t = \sum_x e^{{}_na_x + {}_nb_x k_t} N(x,n,t)$ is thus the total number of deaths (by definition). ${}_na_x$ and ${}_nb_x$ were calculated. $N(x,n,t)$ and $D(t)$ were known from the data at hand. To calculate k(t) through several iterations, Microsoft-Excel system was used.

2.4 Data Interpretation

$$ln[m(x,n,t)] =_n a_x +_n b_x k_t$$

even when $k_t = 0$, $ln[m(x,n,t)] =_n a_x$
The above mentioned model is unique in the sense that the time effect and the age effect are separately represented. And when the time effect has been taken to be negligible ($k_t = 0$), then also the $ln[m(x,n,t)]$ takes the value ${}_na_x$ or $m(x,n,t) = e^{{}_na_x}$. ${}_na_x$: The intrinsic age effect (irrespective of time) is represented by the curve ${}_na_x$. This intrinsic age effect (irrespective of time) falls steadily from 0 to 5-14 (Figure 6 and Figure 8). Intuitively also it is clear that the age factor (during the childhood and old age) has a powerful effect on the mortality. As the age increases, the resistance to illness increases, but as one encroaches senility, the trend is reversed. This trend is very well represented by the graph of ${}_na_x$. As seen from the graph of ${}_na_x$. It rises sharply from 0 to $5-14$ and then decreases steadily.
${}_nb_x$: In the equation, ${}_nb_x$ is the slope of the equation. By definition, slope ${}_nb_x$ is the change in $ln(m(x,n,t))$ with a unit change in k_t. Or in other words,

Table 2.7: Lee and Carter:Calculated and Observed ASDR Female, Nepal 1976-91

Age			1991		1986		1976	
	a_x	b_x	calculated		calculated		calculated	
0	-2.1756	0.0473	0.1018	0.1085	0.1104	0.1046	0.1263	0.1379
1-4	-3.5972	0.1524	0.0193	0.0146	0.0251	0.0358	0.0386	0.0372
5-14	-5.2643	0.0931	0.0042	0.0032	0.0049	0.0066	0.0064	0.0061
15-24	-5.2639	0.098	0.0041	0.0044	0.0049	0.0041	0.0065	0.006
25-34	-5.0024	0.1164	0.0051	0.0061	0.0063	0.0042	0.0087	0.0107
35-44	-4.6031	0.1335	0.0074	0.0078	0.0093	0.0068	0.0135	0.0148
45-54	-3.8654	0.1731	0.0141	0.0115	0.0189	0.021	0.0309	0.0349
55-64	-3.3713	0.1258	0.0257	0.023	0.0319	0.0289	0.0456	0.0481
65+	-2.2881	0.0605	0.0882	0.0812	0.0980	0.0991	0.1163	0.1081
kt			-2.3068		-0.5813		2.2504	

${}_nb_x$ is the sensitivity of mortality to time. As seen from the graph, the mortality condition of age group $1-4$ has reacted most sensitively to time changes (Figure 7 and Figure 9). That is the time has affected the mortality of this age group the most. The mortality of the age group 45-54 and onwards also shows a considerable sensitivity to time. Here time indirectly represents many factors, the development of better medical facilities and hence better health conditions and so on. The more developed the country, the better the medical conditions. In developed world the mortality will be less time sensitive as it has attained already its minimum in comparison to the less developed world. So, for example the b_x values for Germany will be lower compared to that of Nepal.

Table 2.8: Lee and Carter:Calculated and Observed ASDR Male, Nepal 1976-91

Age			1991		1986		1976	
	a_x	b_x	calculated		calculated		calculated	
0	-2.1412	0.0483	0.1060	0.1011	0.1179	0.1112	0.1308	0.1284
1-4	-3.8131	0.2284	0.0135	0.0103	0.0224	0.0201	0.0366	0.0326
5-14	-5.3266	-0.0440	0.0053	0.0068	0.0048	0.0034	0.0044	0.0052
15-24	-5.4606	0.1651	0.003	0.0024	0.0043	0.0043	0.0061	0.006
25-34	-5.1986	0.0781	0.0047	0.0045	0.0056	0.0056	0.0067	0.0073
35-44	-4.8753	0.0828	0.0064	0.0058	0.0077	0.007	0.0092	0.008
45-54	-3.9611	0.1938	0.0126	0.0095	0.0193	0.0451	0.0292	0.0454
55-64	-3.4402	0.1611	0.0227	0.01837	0.0324	0.0342	0.0458	0.0451
65+	-2.2847	0.0863	0.0846	0.0863	0.1024	0.0821	0.1232	0.1345
kt			-2.1413		0.0683		2.2131	

Chapter 3

Model suited to Nepal's need

3.1 Objective

The objective was to formulate a parsimonious model, specially suited to Nepal's needs. In other words, a model with minimum number of parameters has to be constructed. Not only Nepal, but also the most of the countries of the developing world, have the problem of defective data. In such a situation the construction of an ambitious model with large number of parameters and complicated convergence conditions would not be suitable in Nepalese context. So a simple model, which neither involved complicated progamming nor an intensive use of computer softwares was constructed. Inspite of its simplicity, this model gave very good results. Further, the accuracy of the parameters and the data on which the model is based is discussed in the succeeding chapter. There were three criteria, which were judged while selecting an optimal model. Empirical accuracy in explaining a wide variety of observed patterns, economy of parameters and ability of the parameters to be meaningfully interpreted [17].

3.2 Construction of the model

l_x, a very important life table function, is the probability of survival from birth to age x. l_x can be used to construct other life table functions such as μ_x(force of mortality).

$$\mu_x = \frac{-1}{l_x}\frac{dl_x}{dx}$$

$ln(\frac{1}{l_x})$, when plotted against x lie almost on a straight line (Figure 11 and Figure 10).

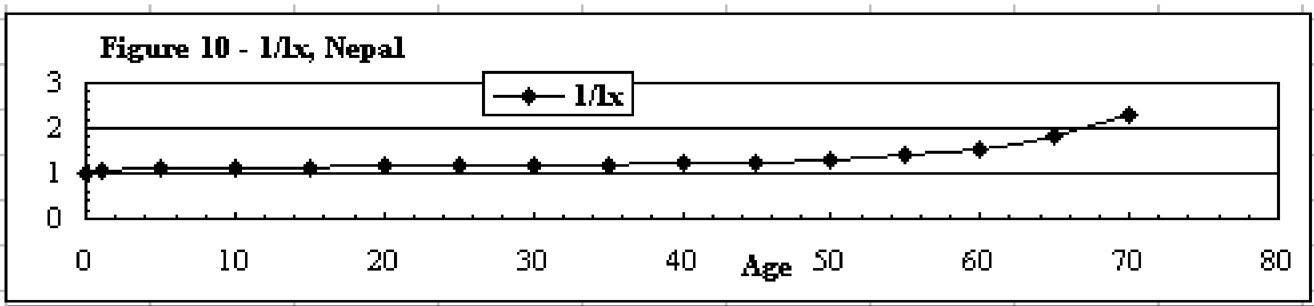


Figure 10 - 1/lx, Nepal

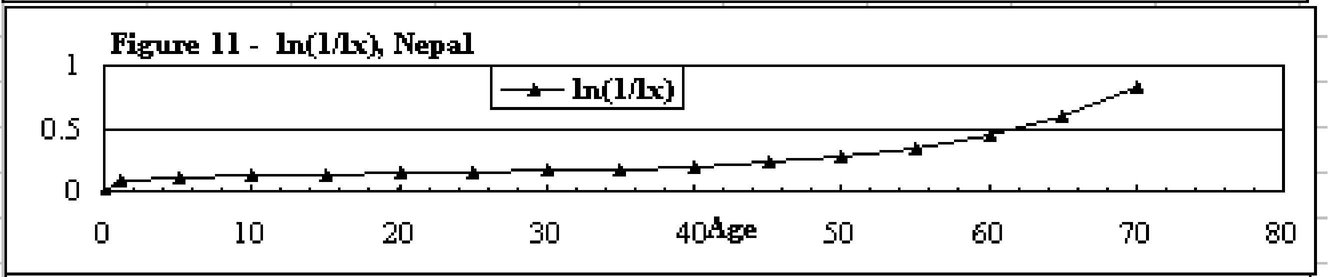


Figure 11 - ln(1/lx), Nepal

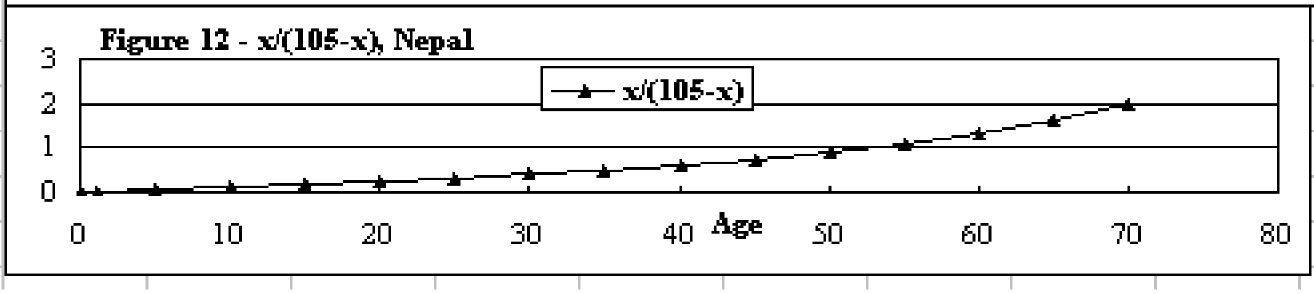


Figure 12 - x/(105-x), Nepal

This linearity of $ln(\frac{1}{l_x})$ was made use of. In addition to that, the age and time effects were seperated. Lee and Carter 1992 [36] was the main motivation behind this decision. Apart from the seperation of age and time effect; the weights $\frac{x}{105-x}$ (the intensity to survive) were considered (Figure 12). Maximum longivity was assumed to be 105 years.

$$y = \frac{x}{105 - x}$$

$$\frac{dy}{dx} = \frac{105}{(105 - x)^2}$$

$$\frac{dy}{dx} > 0$$

$$\frac{d^2y}{dx^2} > 0$$

$\frac{x}{105-x}$ (the intensity to survive) increases as x (age) increases. $\frac{dy}{dx} > 0$; but as $\frac{d^2y}{dx^2} < 0$, the change in the intensity to survive increases with age.

3.3 The Model

$$\frac{1}{l_x} = e^{[a_x + b_x.k_t]\frac{x}{105-x}} \tag{3.1}$$

Where, l_x is the probability of survival and $0 < l_x < 1$. This implies that $1 < \frac{1}{l_x} < \infty$

Substituting $x = 0$ and $x = 105$ in equation (3.1) we get,

$$\frac{1}{l_x} = 1$$

and

$$\frac{1}{l_x} = \infty$$

respectively. Thus the factor $\frac{x}{105-x}$ keeps l_x in the model (3.1) within bounds 0 and 1.

$$ln(\frac{1}{l_x}) = a_x.\frac{x}{105 - x} + b_x.\frac{x}{105 - x}.k_t$$

$$ln(\frac{1}{l_x}) = [a_x + b_x.k_t]\frac{x}{105 - x} + e_x \tag{3.2}$$

$$(\frac{105 - x}{x})ln(\frac{1}{l_x}) = a_x + b_x.k_t + e_x \tag{3.3}$$

Equation (3.2) is not a simple linear equation as all the terms on the R.H.S are unknown. Besides, the parameters a_x, b_x, k_t do not have unique values.

Let the solution of (3.2) be $a_x = a$; $b_x = b$; $k_t = k$ so,

$$ln(\frac{1}{l_x}) = [a + bk]\frac{x}{105 - x} + e_x$$

or,

$$ln(\frac{1}{l_x}) - [a + bk]\frac{x}{105 - x} - e_x = 0$$

if $a_x = a - bc$, $b_x = b$, $k_t = k + c$

$$ln(\frac{1}{l_x}) = [a - bc + bk + bc]\frac{x}{105 - x} + e_x$$

$$ln(\frac{1}{l_x}) = [a + bk]\frac{x}{105 - x} + e_x$$

If a, b, k satisfies (3.2) then $a - bc$, b, $k + c$ is also a solution of (3.2). Thus (3.2) does not have unique solution. To get distinct solution of (3.2), following condition has been imposed.

$$\sum_x b_x = 1$$

$$\sum_t k_t = 0$$

Demoivre in 1725 hypothesized a linear relationship between age x and survivorship function of the life table l_x [17].

$$l_x = l_o(1 - \frac{x}{\alpha})$$

where α is an upper age limit. This model has lead to the choice of the factor $\frac{x}{(105-x)}$.

3.4 Model construction

3.4.1 Steps in the construction of the model

1. Logarithmic Transformation: The data is available in the form l_x and then its reciprocal is subjected to the logarithmic transformation.

2. Calculation of a_x

$$\frac{1}{l_{x,t}} = e^{[a_x + b_x . k_t]\frac{x}{105-x}}$$

$$ln(\frac{1}{l_{x,t}}) = a_x.\frac{x}{105 - x} + b_x.\frac{x}{105 - x}.k(t)$$

$$\sum_t ln(\frac{1}{l_{x,t}}) - \frac{x}{105-x}.\sum_t a_x = \frac{x}{105-x}.\sum_t b_x.k_t$$

$$\sum_t k_t = 0$$

$$a_x = (\frac{105-x}{x})\frac{\sum_t ln(\frac{1}{l(x)})}{\sum_t 1}$$

3. Calculation of k_t

$$\sum_x ln(\frac{1}{l_{x,t}}) - \frac{x}{105-x}\sum_x a_x = \frac{x}{105-x}.\sum_x b_x k_t$$

as,

$$\sum_x {}_n b_x = 1$$

$$k_t = \sum_x^{\frac{105-x}{x}} \ln(\frac{1}{l_{x,t}}) - \sum_x a_x$$

4. Calculation of b_x The model was reduced to

$$ln(\frac{1}{l_{x,t}}) - a_x\frac{x}{105-x} = b_x k_t\frac{x}{105-x} + e_{x,t}$$

Now a_x and the initial values of k_t are known.

The next step is taken towards the estimation of the parameter b_x. As the model was not a proper regression equation, without a proper independent and dependent variable and k_t was not a full rank non singular matrix, singular value decomposition comes to the rescue.

Now with the help of singular value decomposition the value of b_x is found.

$y = bk$ is a system of linear equations, where k is a singular matrix.

$$yk^+ = bkk^+$$

$$yk^+ = b$$

where k^+ is the pseudo-inverse of k. For a general matrix A, the singular value decomposition of A.

$$A = USV^T$$

where U and V are orthogonal matrices and S is a diagonal matrix. Let A^+ be the pseudo-inverse of A

$$A^+ = (USV^T)^+ = (V^T)^+ S^+ U^+ \tag{3.4}$$

because the matrices V^T and U are orthogonal, their pseudo-inverses are simply their ordinary inverses.

$$(V^T)^+ = V$$
$$U^+ = U^T$$

the pseudo inverse of a rectangular diagonal matrix S has to be defined. Let D be an $m \times n$ diagonal matrix. $D = diag(d_i)$. The pseudo-inverse of D, denoted by $D+$, is an $n \times m$ diagonal matrix $diag({d_i}^+)$ whose i - th element ${d_i}^+$ is given by

$${d_i}^+ = \begin{cases} 1/d_i & : \quad if d_i \neq 0 \\ 0 & : \quad if d_i = 0 \end{cases}$$

The dimension of D^+ are the reversed dimension of D ie if D is $m \times n$ D^+ is $n \times m$. A non zero element in D corresponds to its reciprocal in D^+, but a zero diagonal element in D is associated with a zero diagonal element in D^+. The pseudo-inverse of k_t of equation was calculated using Mathematica. The method is explained in detail on pages 32 and 32.

3.5 Result

R^2 is the variance explained by regression. If T.S.S is the total sum of squares then,

$$T.S.S = \sum_i (y_i - \overline{y})^2$$

where

$$\overline{y} = \sum_{i=1}^{n} y_i$$

S.S.R is the sum of squares due to regression. S.S.E is the sum of squares due to error.

$$S.S.R = \sum_i (\hat{y}_i - \overline{y})^2$$

$$S.S.E = \sum_i (y_i - \widehat{y}_i)^2$$
$$T.S.S = S.S.R + S.S.E$$

T.S.S is the total variation.
S.S.R is the variation explained by regression.
S.S.E is the unexplained variance.
R^2 =proportion of variation explained by regression.
$R^2 = \frac{S.S.R}{T.S.S}$

$$M.S.E = \frac{S.S.E}{error d.f}$$

If y_i,such that $i = 1, \cdots, n$, total degree of freedom is $n-1$ degree of freedom due to regression is 3(no of parameters)-1 ie 2, degree of freedom due to error is n-2 [59].

Seeing the values of R^2 and $M.S.E$, it could be concluded that the model had shown very good results for Nepal as well as for India and Germany. This model fitted equally well for Nepal (with deficient and defective data) and Germany (with abundant and accurate data)(Figures 21-32).

$${}_nd_x = l_x - l_{x+n}$$
$${}_nq_x = \frac{{}_nd_x}{l_x}$$

When l_x can be projected, ${}_nd_x$ and ${}_nq_x$ follow l_x by the formula above.

3.6 Interpretation of the parameters

As seen from Figures 13-18,
a_x:

It is the average of the log of reciprocal of probability of survival. It is known that $l_x(Nepal) < l_x(India) < l_x(Germany)$ and thus $\frac{1}{l_x}(Germany) < \frac{1}{l_x}(India) < \frac{1}{l_x}(Nepal)$ This implies further that
$ln(\frac{1}{l_x}(Germany)) < ln(\frac{1}{l_x}(India)) < ln(\frac{1}{l_x}(Nepal))$ So as seen from the comparitive graph a_x for Nepal is higher than a_x for India, which in turn is higher for Germany (Figure 20).

b_x:

Similarly as seen from equation (3.2), it is the relative sensivity of probability of survival to variation in k. The younger the age the greater is the sensitivity to variation in k. The sensitivity at younger ages is highest for Nepal as compared to India and Germany. This can be seen from the graph.

l_x for Germany is the least time sensitive, but l_x for Indian females is the most time sensitive (Figure 19).

$$\frac{dln(\frac{1}{l_x})}{dt} = b_x \frac{x}{105 - x} \frac{dk_t}{dt}$$

In our case b_x is positive. This implies that the probability of survival for that age decreases as $\frac{1}{l_x}$ increases. Further the smoothness in the movement of b_x is not exhibited by Nepal in comparison to Germany and India. This perhaps reflects the inaccuracy of the data in the respective age groups. If **k** declines linearly with time, then $\frac{dk_t}{dt}$ from above will be a constant and thus $\frac{1}{l_x}$ will increase at exponential rate. k_t is the intensity and decreases smoothly at a roughly constant rate, with time, as seen from the graph of k_t.

3.7 Advantages

1. Simplicity; The model is very simple. Inspite of its simplicity, the model has shown very good results.
2. This model does not require complicated software programmes.
3. The model is very well suited to Nepalese needs. It has shown very good results not only for Nepal, which has limited and defective data, but also for Germany with highly accurate and abundant data.

3.8 Projection

India, Nepal: As k_t values lay roughly on a straight line, the value of k_t was linearly extrapolated. So, for given a_x and b_x and extrapolated time effect k_t; l_x was projected. Thus from three sets of l_x values for years 1976, 1986 and 1991 l_x values ranging from the year 1945 to 2001 could be projected using this model. Similarly a projection was made for India. The accuracy of the projected l_x values for India were judged by comparing with e_0. In the case of Nepal and India linear extrapolations were made, that is a straight line was fitted to the parameter with respect to time and the value of the parameter was estimated for a time point in future with the help of the linear equation. This was done considering the drastic improvement in the life expectation in Nepal and India. For Germany on the other hand, instead of linear extrapolation logistic extrapolation was used. Germany, like other developed countries, has already achieved a low mortality several years ago. Thus, the rate of change of mortality in Germany is much lower.

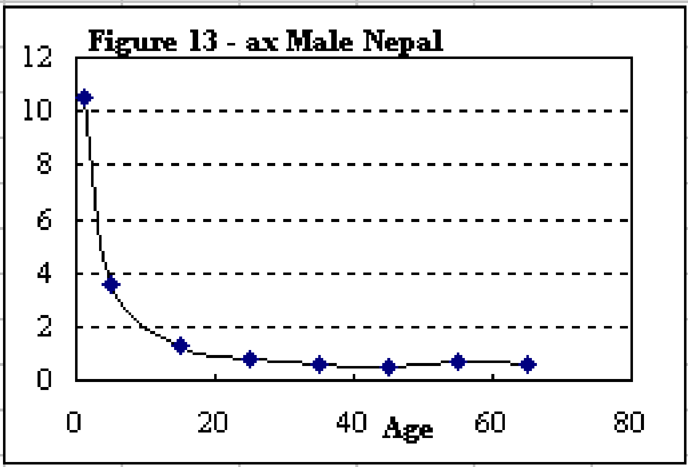


Figure 13 - ax Male Nepal

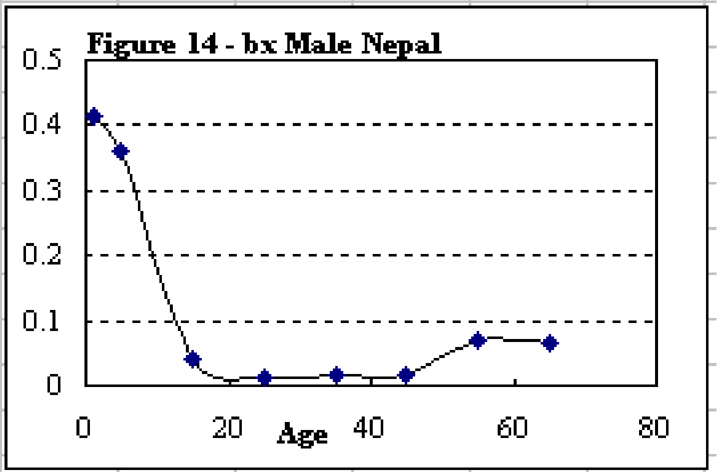


Figure 14 - bx Male Nepal

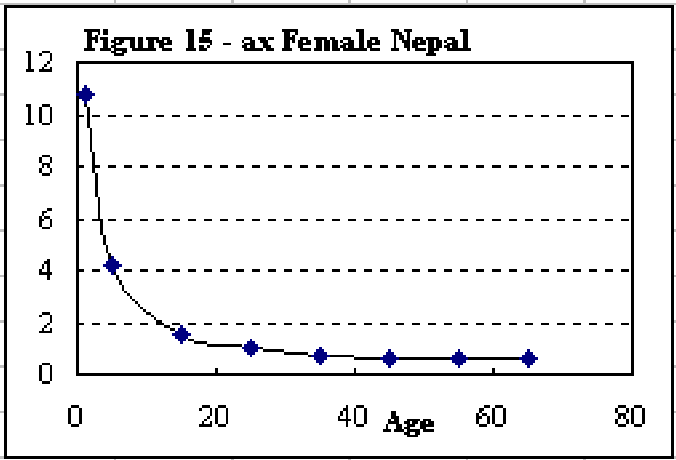


Figure 15 - ax Female Nepal

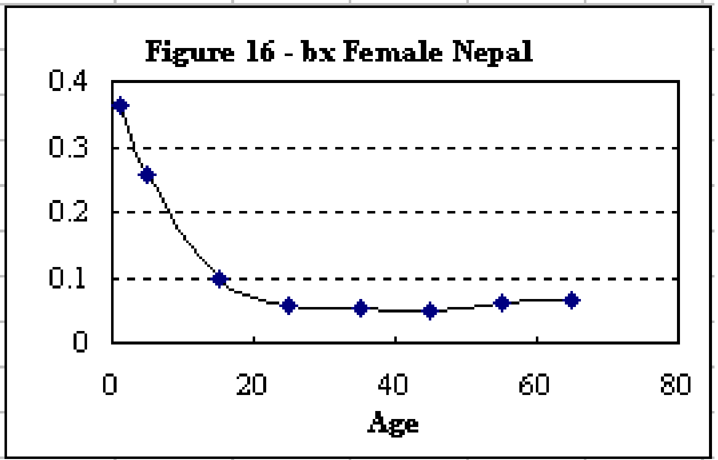


Figure 16 - bx Female Nepal

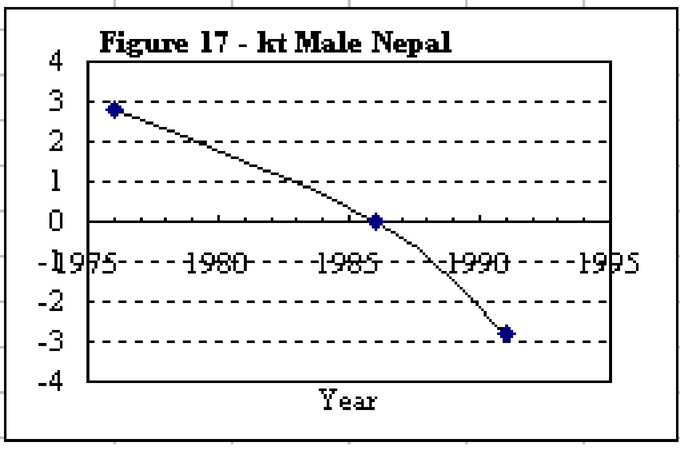


Figure 17 - kt Male Nepal

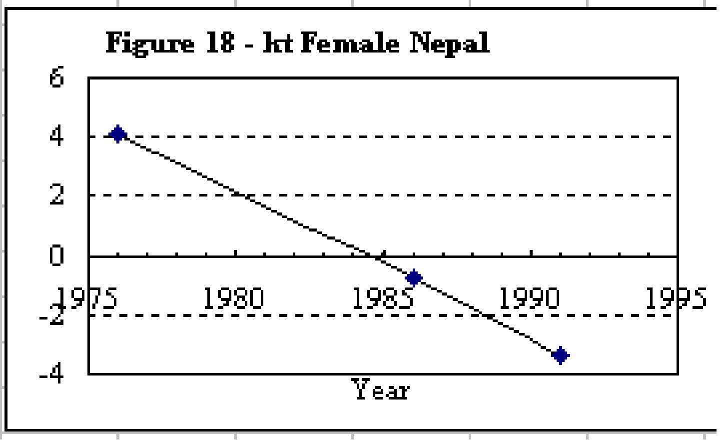


Figure 18 - kt Female Nepal

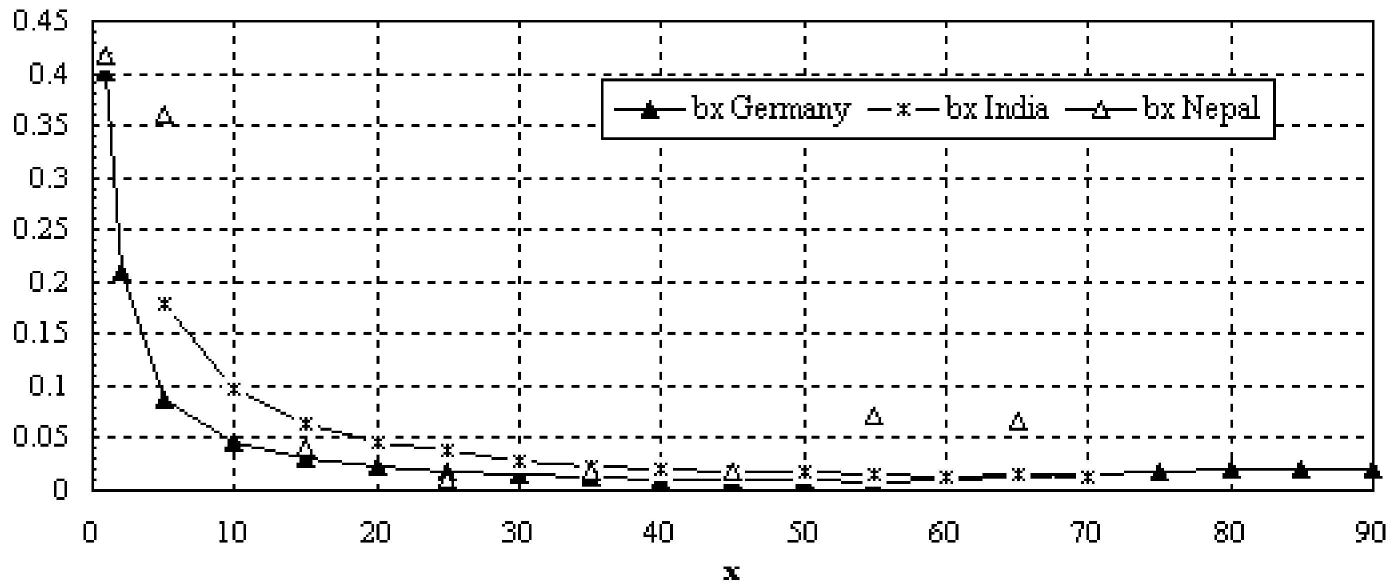


Figure 19 - Comparison of bx between Germany, India and Nepal

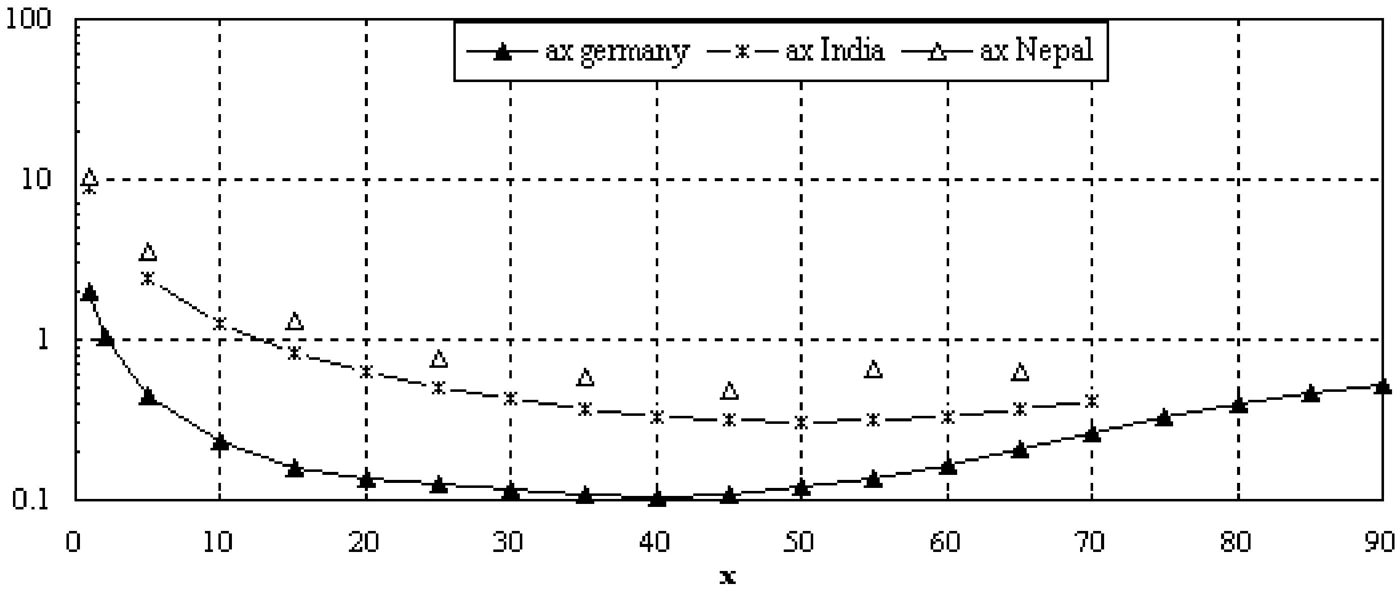


Figure 20 - Comparison of ax between Germany, India and Nepal

Figure 21 - Comparison between observed and predicted lx; Nepal Male, 1976-1991

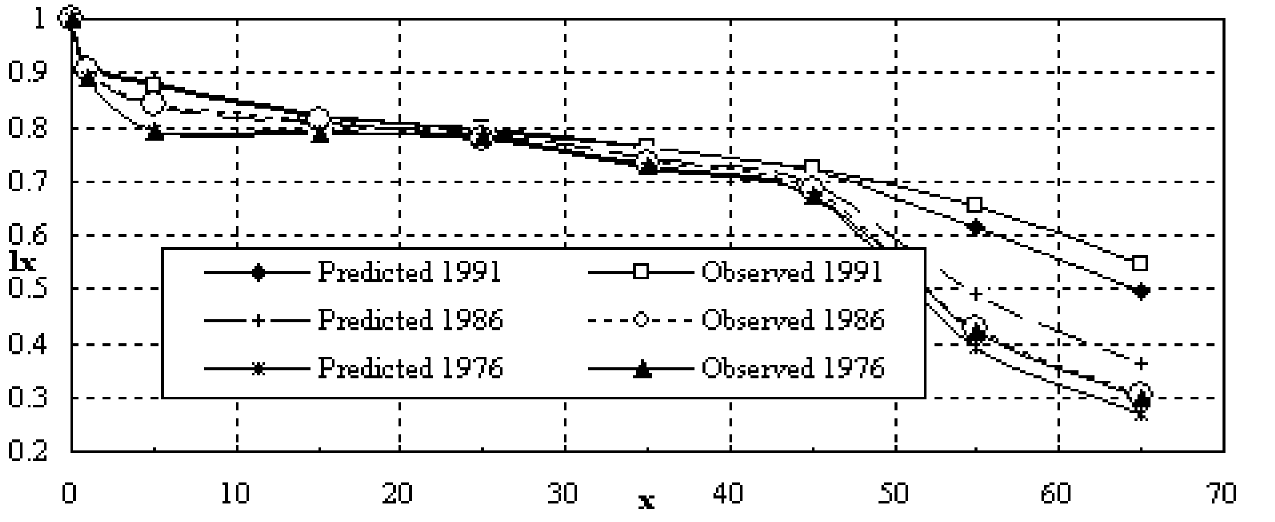


Figure 22 - Comparison between observed and predicted lx; Nepal Female, 1976-1991

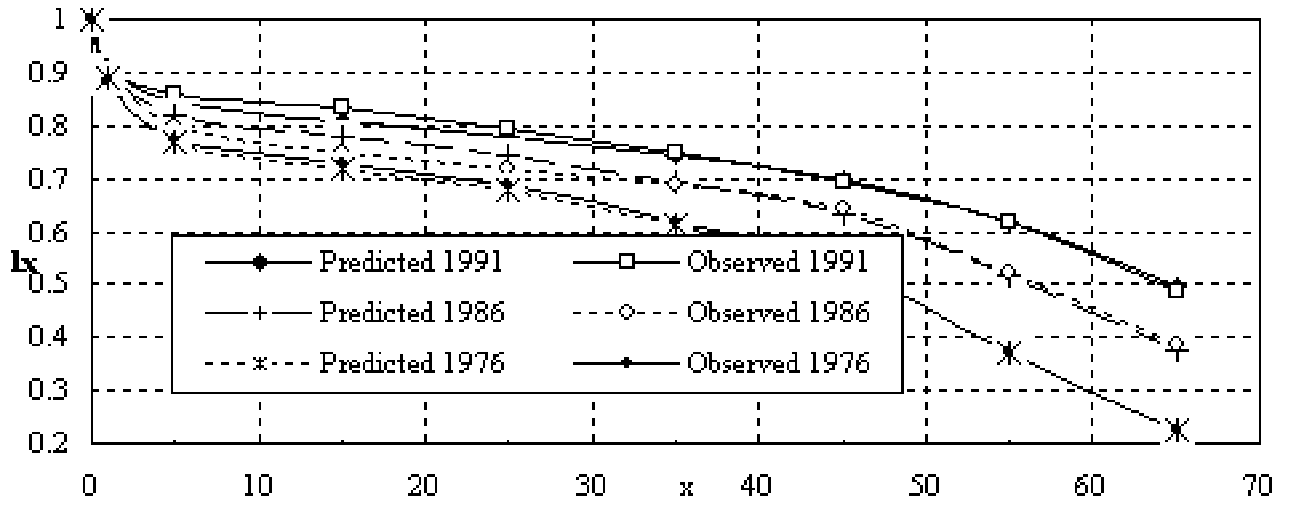


Figure 23 - Comparison between rural and urban lx, female 1991-95, India

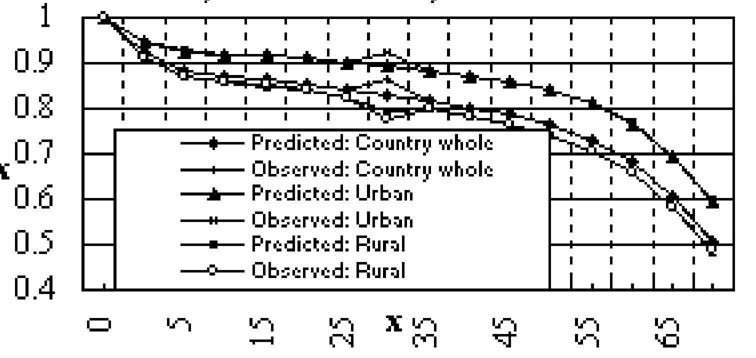


Figure 24 - Comparison between rural and urban lx, male 1991-95, India

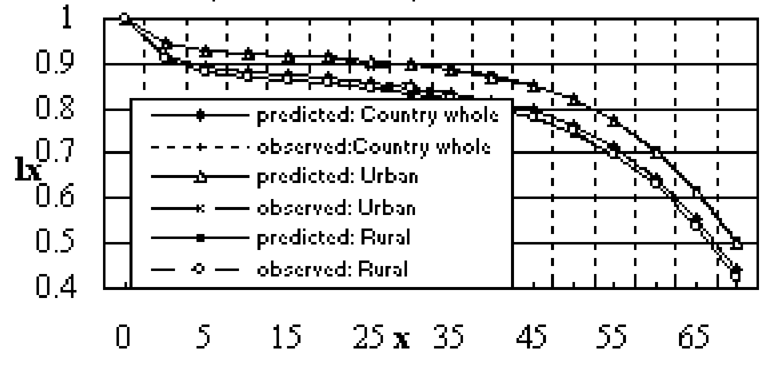


Figure 25 - Comparison between rural and urban lx, female 1988-92, India

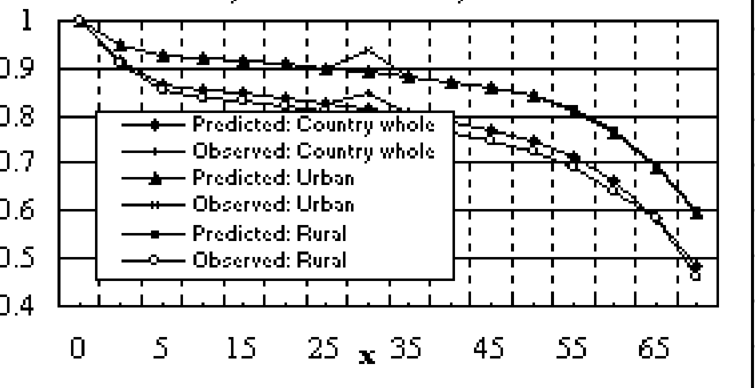


Figure 26 - Comparison between rural and urban lx, Male 1988-92, India

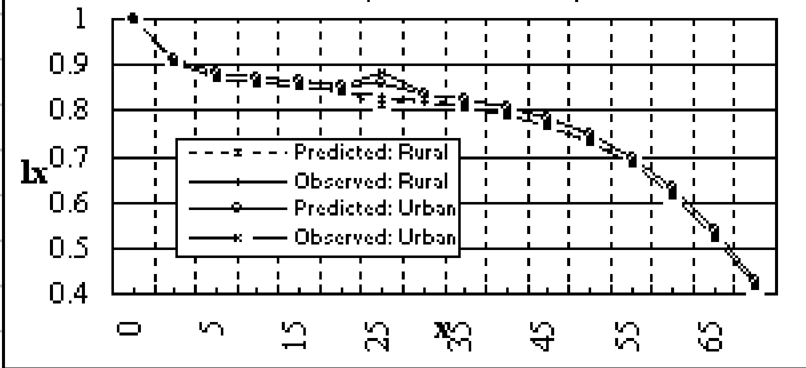


Figure 27 - Comparison between observed and predicted lx, Male 1988-92, India

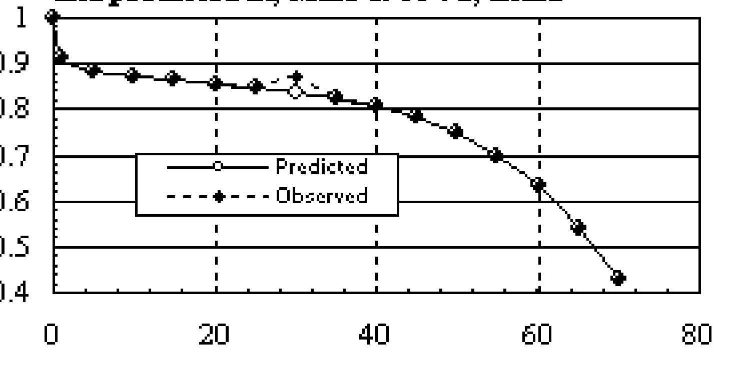
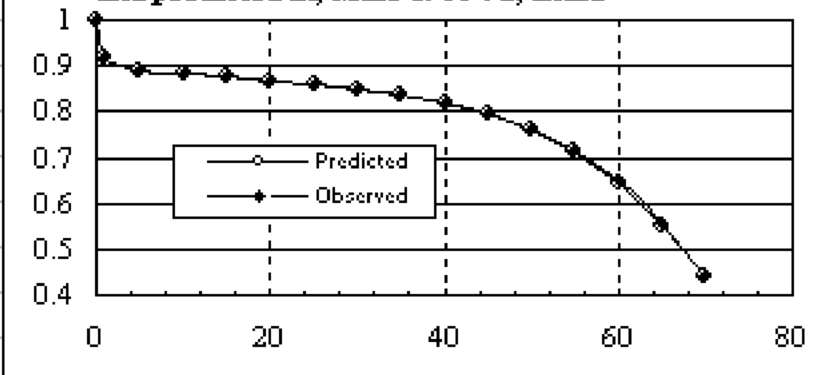


Figure 28 - Comparison between observed and predicted lx, Male 1988-92, India



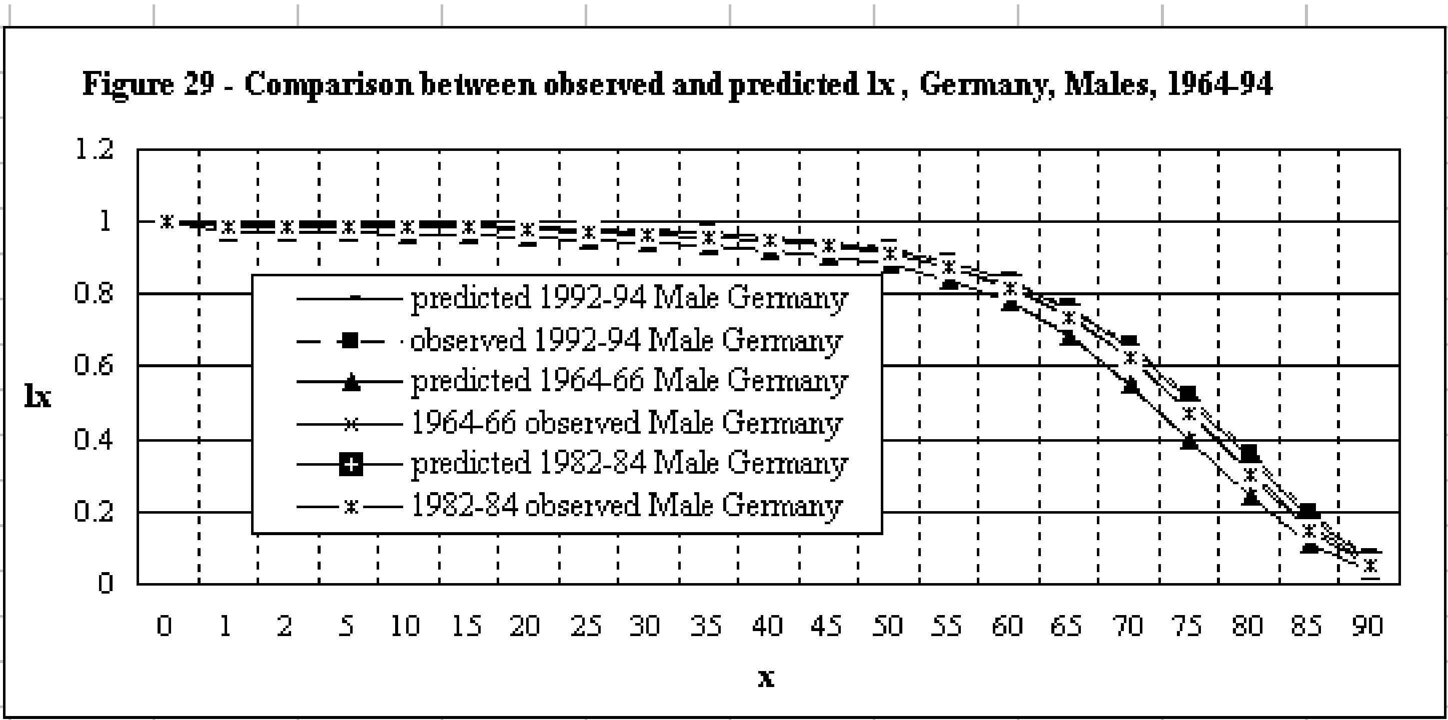


Figure 29 - Comparison between observed and predicted lx , Germany, Males, 1964-94
lx
1.2
1
0.8
0.6
0.4
0.2
0
predicted 1992-94 Male Germany
observed 1992-94 Male Germany
predicted 1964-66 Male Germany
1964-66 observed Male Germany
predicted 1982-84 Male Germany
1982-84 observed Male Germany
0 1 2 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90
x

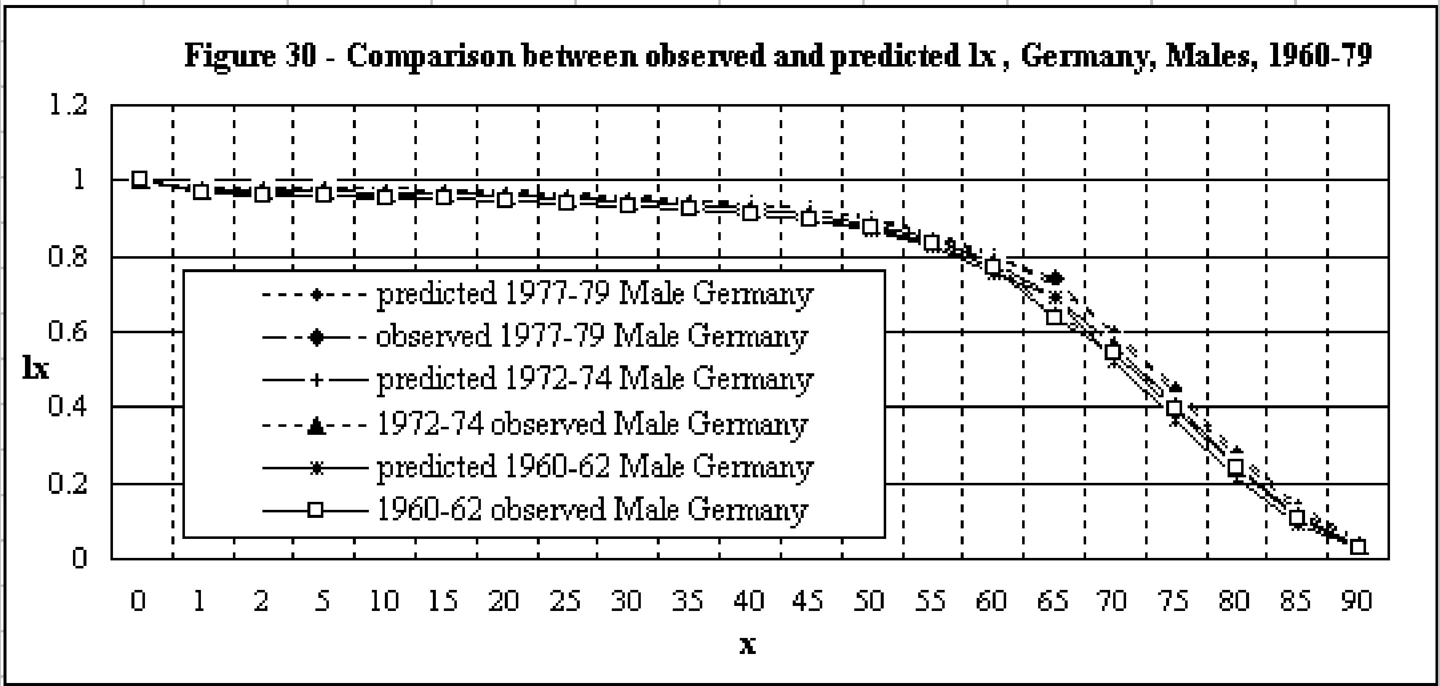


Figure 30 - Comparison between observed and predicted lx , Germany, Males, 1960-79
lx
1.2
1
0.8
0.6
0.4
0.2
0
predicted 1977-79 Male Germany
observed 1977-79 Male Germany
predicted 1972-74 Male Germany
1972-74 observed Male Germany
predicted 1960-62 Male Germany
1960-62 observed Male Germany
0 1 2 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90
x

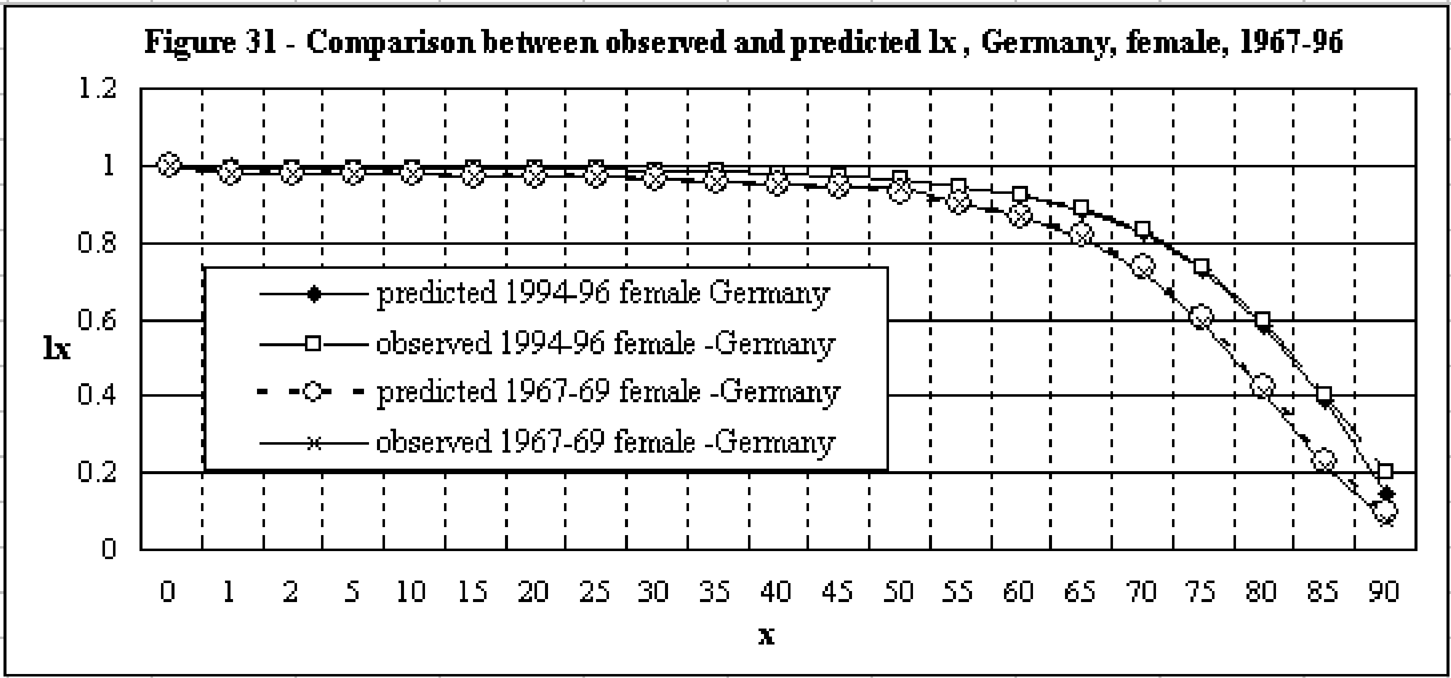


Figure 31 - Comparison between observed and predicted lx , Germany, female, 1967-96
1.2
1
0.8
0.6
0.4
0.2
0
lx
predicted 1994-96 female Germany
observed 1994-96 female -Germany
predicted 1967-69 female -Germany
observed 1967-69 female -Germany
0 1 2 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90
x

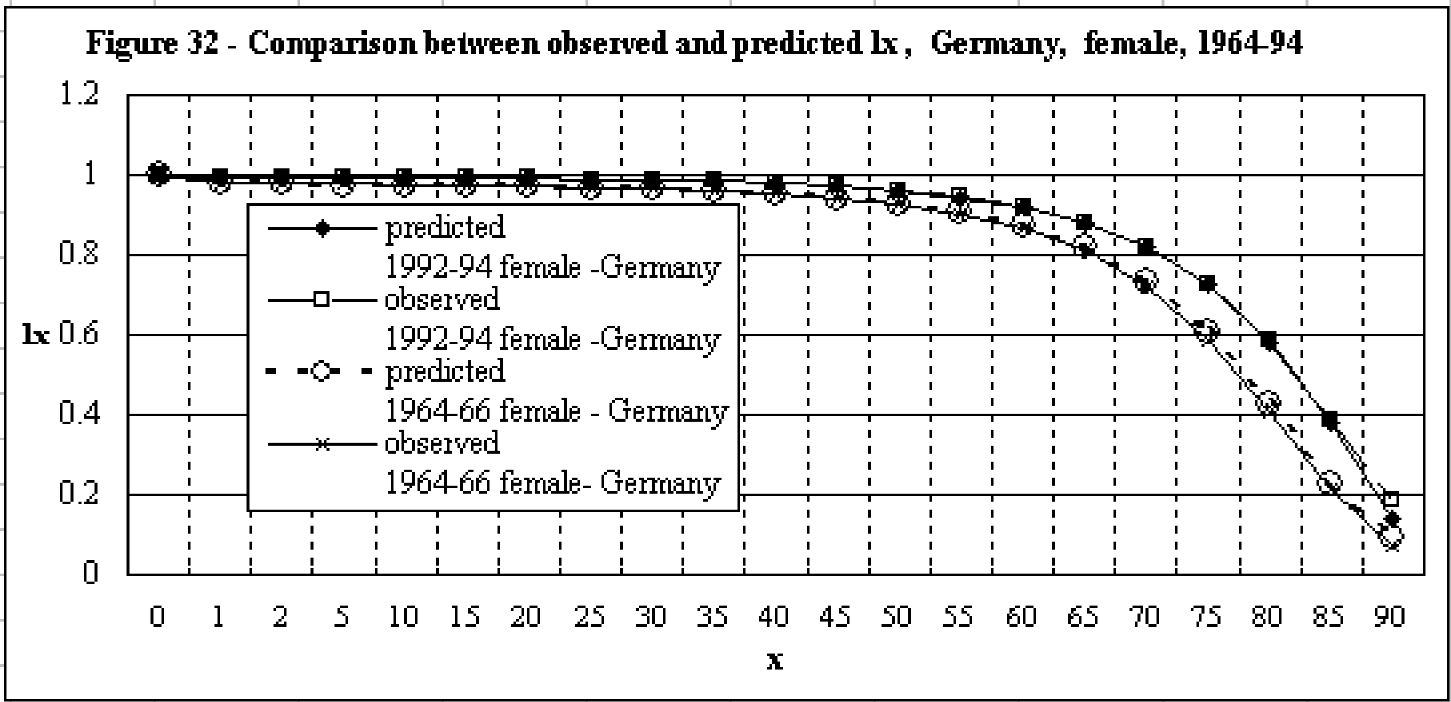


Figure 32 - Comparison between observed and predicted lx , Germany, female, 1964-94
1.2
1
0.8
0.6
0.4
0.2
0
lx
predicted
1992-94 female -Germany
observed
1992-94 female -Germany
predicted
1964-66 female - Germany
observed
1964-66 female- Germany
0 1 2 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90
x

Table 3.1: Observed and Predicted l_x, Nepal 1971-1991, Male

Age	1991		1986		1976	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
0	1	1	1	1	1	1
1	0.9140	0.9132	0.9038	0.9055	0.8936	0.892795
5	0.8797	0.8778	0.8361	0.8397	0.7943	0.7926
15	0.8223	0.8198	0.8066	0.8115	0.7911	0.7887
25	0.7949	0.7999	0.7871	0.7773	0.7793	0.7842
35	0.7608	0.7649	0.7427	0.7348	0.7248	0.7287
45	0.7175	0.7216	0.6927	0.6848	0.6685	0.6723
55	0.6125	0.6557	0.4914	0.4290	0.3936	0.4212
65	0.4944	0.5442	0.36543	0.3019	0.2695	0.2964

Table 3.2: Observed and Predicted l_x, Nepal 1971-1991, Female

Age	1991		1986		1976	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
0	1	1	1	1	1	1
1	0.9120	0.9076	0.9037	0.91051	0.8883	0.88598
5	0.8458	0.8585	0.8182	0.7997	0.7684	0.77455
15	0.8111	0.8318	0.7778	0.7484	0.7188	0.7285
25	0.7786	0.7955	0.7422	0.71822	0.6781	0.6898
35	0.7433	0.7482	0.6955	0.68858	0.6136	0.61572
45	0.6974	0.6917	0.6351	0.64308	0.5324	0.53009
55	0.6177	0.6161	0.5175	0.51953	0.3707	0.37023
65	0.4952	0.4877	0.3776	0.38655	0.2265	0.22467

Germany: The model was based on a large sets (9) of l_x values ranging from the time period 1964-66 to 1994-96, and a comparitively longer range of projection into the future could be made, in comparison to Nepal and India. The former could be a plausible reason for the later. Here a logistic curve was used for extrapolating k_t into the future.

Seeing the values of R^2 and M.S.E, it can be concluded that the projected l_x values for Nepal, India and Germany are reliable.

Logistic extrapolation:

In the case of Germany, logistic extrapolation has been made. The

Table 3.3: Observed and Predicted l_x, India, Male,1988-91

Age	1991		1990		1988	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
0	1	1	1	1	1	1
1	0.9196	0.9192	0.9196	0.92002	0.9150	0.91504
5	0.8910	0.8916	0.8911	0.89058	0.8812	0.8812
10	0.8813	0.8819	0.8814	0.8808	0.8701	0.8701
15	0.8756	0.8762	0.8757	0.87513	0.8640	0.86402
20	0.8681	0.8687	0.8683	0.8677	0.8565	0.8565
25	0.8587	0.85917	0.8589	0.8584	0.8463	0.84632
30	0.8477	0.84811	0.8479	0.8475	0.8355	0.8355
35	0.8346	0.835	0.8347	0.8344	0.8226	0.8226
40	0.8184	0.8189	0.8186	0.8181	0.8059	0.8059
45	0.7965	0.79694	0.7967	0.7963	0.7831	0.7831
50	0.7627	0.7634	0.7628	0.7621	0.7483	0.7483
55	0.7145	0.7157	0.7147	0.71351	0.6997	0.69972
60	0.6458	0.64801	0.6459	0.64382	0.6311	0.63105
65	0.5526	0.55509	0.5528	0.55029	0.5370	0.53698
70	0.4389	0.44232	0.4390	0.43569	0.4263	0.42626

Table 3.4: Observed and Predicted l_x, India, Urban male

Age	1991		1990		1988	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
0	1	1	1	1	1	1
1	0.9455	0.9452	0.9196	0.9200	0.9150	0.91504
5	0.9291	0.9294	0.8911	0.8906	0.8812	0.88121
10	0.9235	0.9236	0.8814	0.8808	0.8701	0.8701
15	0.9194	0.9196	0.8757	0.87513	0.8640	0.86402
20	0.9139	0.9143	0.8683	0.8677	0.8565	0.85645
25	0.9063	0.9065	0.8589	0.85844	0.8463	0.8463
30	0.8972	0.8976	0.8479	0.84748	0.8355	0.83547
35	0.8857	0.8865	0.8347	0.8344	0.8226	0.8226
40	0.8716	0.8726	0.8186	0.8181	0.8059	0.80591
45	0.8511	0.85197	0.7967	0.7963	0.7831	0.78313
50	0.8190	0.82045	0.7628	0.76212	0.7483	0.74832
55	0.7723	0.7744	0.7147	0.7135	0.6997	0.69972
60	0.7050	0.709	0.6460	0.64382	0.6311	0.63105
65	0.6138	0.61734	0.5528	0.55029	0.5370	0.53698
70	0.5009	0.50714	0.4390	0.4357	0.4263	0.42626

Table 3.5: Observed and Predicted l_x, India, Rural male

Age	1991		1990		1988	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
0	1	1	1	1	1	1
1	0.9135	0.91303	0.9137	0.9142	0.9087	0.90874
5	0.8819	0.88256	0.8822	0.8815	0.8714	0.8714
10	0.8709	0.8717	0.8712	0.8704	0.8590	0.85898
15	0.8647	0.8666	0.8651	0.8643	0.8524	0.85235
20	0.8566	0.8575	0.8570	0.8562	0.8444	0.84433
25	0.8466	0.8473	0.8470	0.84634	0.8336	0.8336
30	0.8349	0.8355	0.8353	0.8347	0.8223	0.8223
35	0.8210	0.8216	0.8214	0.8208	0.8088	0.80883
40	0.804	0.8047	0.8043	0.8037	0.7916	0.79154
45	0.7814	0.7821	0.7818	0.78104	0.7678	0.7678
50	0.7467	0.7479	0.7471	0.74595	0.7325	0.73251
55	0.6980	0.6995	0.6984	0.69694	0.6845	0.6845
60	0.6289	0.63133	0.6293	0.62691	0.6176	0.61751
65	0.5358	0.5383	0.5361	0.5338	0.5246	0.5246
70	0.4223	0.42554	0.4225	0.4194	0.4181	0.418

Table 3.6: Observed and Predicted l_x, India, Female, 1988-91

Age	1991		1990		1988	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
0	1	1	1	1	1	1
1	0.9211	0.9207	0.9212	0.92153	0.9172	0.91721
5	0.8807	0.88145	0.8809	0.88016	0.8681	0.86813
10	0.8688	0.86955	0.8690	0.86823	0.8546	0.85461
15	0.8624	0.86314	0.8626	0.86179	0.8479	0.84784
20	0.8525	0.85352	0.85272	0.85172	0.8370	0.83701
25	0.8405	0.84077	0.8407	0.84051	0.8255	0.8255
30	0.8280	0.828	0.8282	0.82816	0.8133	0.81333
35	0.8154	0.81498	0.8156	0.81596	0.8013	0.80134
40	0.8019	0.80164	0.8021	0.80243	0.7875	0.78747
45	0.7857	0.78553	0.7859	0.78599	0.7708	0.77079
50	0.7639	0.76416	0.7641	0.7638	0.7482	0.74822
55	0.7299	0.73041	0.7301	0.72959	0.7138	0.7138
60	0.6812	0.68268	0.6815	0.68	0.6632	0.66321
65	0.6056	0.60761	0.6059	0.60388	0.5877	0.58764
70	0.5059	0.51038	0.5062	0.50183	0.4839	0.48386

Table 3.7: Observed and Predicted l_x, India, Urban - female, 1988-91

Age	1991		1990		1988	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
0	1	1	1	1	1	1
1	0.9485	0.94832	0.9485	0.9491	0.9485	0.94832
5	0.9263	0.92653	0.9263	0.92667	0.9263	0.92653
10	0.9201	0.92053	0.9201	0.92062	0.9201	0.92053
15	0.9156	0.91603	0.9156	0.91598	0.9156	0.91603
20	0.9090	0.90946	0.9090	0.9095	0.9090	0.90946
25	0.9008	0.9005	0.9008	0.90185	0.9008	0.9005
30	0.8919	0.89149	0.8919	0.89301	0.8919	0.89149
35	0.8824	0.88165	0.8824	0.88341	0.8824	0.88165
40	0.8718	0.871	0.8718	0.87309	0.8718	0.871
45	0.8590	0.85846	0.8590	0.86022	0.8590	0.85846
50	0.8412	0.84191	0.8412	0.84178	0.8412	0.84191
55	0.8121	0.81291	0.8121	0.81344	0.8121	0.81291
60	0.7658	0.76873	0.7658	0.76664	0.7658	0.76873
65	0.6921	0.6955	0.6921	0.69298	0.6921	0.6955
70	0.5936	0.60069	0.5936	0.59311	0.5936	0.60069

Table 3.8: Observed and Predicted l_x, India, Rural - female, 1988-91

Age	1991		1990		1988	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
0	1	1	1	1	1	1
1	0.9147	0.91416	0.9148	0.9153	0.9094	0.90937
5	0.8698	0.87069	0.8700	0.86913	0.8549	0.85489
10	0.8562	0.85717	0.8565	0.8555	0.8399	0.83985
15	0.8492	0.85021	0.8495	0.84852	0.8325	0.83245
20	0.8384	0.83969	0.8387	0.83743	0.8206	0.82063
25	0.8253	0.82574	0.8255	0.82508	0.8081	0.80805
30	0.8115	0.8117	0.8117	0.81154	0.7947	0.79474
35	0.7977	0.79758	0.7979	0.79798	0.7816	0.78158
40	0.7833	0.78335	0.7835	0.78343	0.7669	0.76694
45	0.7657	0.7661	0.76596	0.76557	0.7492	0.7492
50	0.74263	0.74344	0.7429	0.7421	0.7261	0.72605
55	0.7075	0.7085	0.7078	0.70684	0.6914	0.69138
60	0.6582	0.66011	0.6584	0.65651	0.6416	0.6416
65	0.5823	0.58475	0.5823	0.57991	0.5863	0.58622
70	0.4824	0.4875	0.4827	0.47777	0.4638	0.46376

Table 3.9: Observed and Predicted l_x, Germany, Male, 1982-96

Age	1994-96		1992-94		1982-84	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
0	1	1	1	1	1	1
1	0.9949	0.9941	0.994	0.99348	0.9884	0.98865
2	0.9944	0.99358	0.9934	0.99294	0.9875	0.98776
5	0.9935	0.99276	0.9925	0.99199	0.9861	0.98639
10	0.9926	0.99195	0.9915	0.99105	0.9846	0.98489
15	0.9916	0.99105	0.9904	0.99008	0.9832	0.98356
20	0.9872	0.98735	0.9860	0.98635	0.9783	0.97863
25	0.9818	0.98238	0.9805	0.98115	0.9721	0.9724
30	0.9767	0.97753	0.9753	0.97598	0.9664	0.96676
35	0.9703	0.97123	0.9688	0.96913	0.9595	0.96027
40	0.9609	0.96203	0.9593	0.95934	0.94956	0.95097
45	0.9464	0.94811	0.9447	0.94525	0.9345	0.93628
50	0.9241	0.9277	0.9223	0.9242	0.9113	0.91283
55	0.8903	0.89638	0.8882	0.89178	0.8756	0.87494
60	0.8402	0.84891	0.8376	0.84166	0.8215	0.81931
65	0.7652	0.77551	0.7618	0.76515	0.7403	0.73661
70	0.6597	0.6728	0.6551	0.6623	0.6269	0.62348
75	0.5159	0.53839	0.5106	0.5252	0.4788	0.47264
80	0.3478	0.3783	0.3427	0.36371	0.3124	0.29992
85	0.1850	0.21034	0.1812	0.19839	0.1588	0.1468
90	0.0707	0.082	0.0686	0.07572	0.0567	0.05073

Table 3.10: Observed and Predicted l_x, Germany, Male, 1972-81

Age	1979-81		1977-79		1972-74	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
0	1	1	1	1	1	1
1	0.9853	0.98583	0.9830	0.98363	0.9749	0.97502
2	0.9843	0.98487	0.9818	0.98252	0.9733	0.97373
5	0.9826	0.98319	0.98	0.98062	0.9708	0.97138
10	0.9808	0.98125	0.9780	0.97847	0.9681	0.9687
15	0.9793	0.97964	0.9763	0.97672	0.9660	0.96661
20	0.9741	0.97392	0.9710	0.97039	0.9599	0.95976
25	0.9675	0.96682	0.9641	0.96265	0.9522	0.95183
30	0.9615	0.96058	0.9579	0.9562	0.9451	0.94504
35	0.9543	0.95354	0.9505	0.94858	0.9371	0.9369
40	0.9442	0.94345	0.9402	0.93795	0.9262	0.92559
45	0.9289	0.92748	0.9247	0.92162	0.9102	0.90849
50	0.9052	0.90197	0.9007	0.89533	0.88450	0.88271
55	0.8687	0.86331	0.8636	0.85689	0.8458	0.84264
60	0.8127	0.80547	0.8062	0.79858	0.7837	0.78278
65	0.7286	0.72243	0.7200	0.71447	0.6906	0.69194
70	0.6117	0.60476	0.6007	0.59365	0.5633	0.56155
75	0.462	0.4492	0.4498	0.4358	0.4095	0.40076
80	0.2968	0.27874	0.2856	0.26751	0.24958	0.23911
85	0.1476	0.13376	0.1398	0.12727	0.1154	0.10993
90	0.0509	0.04588	0.04709	0.04322	0.0357	0.03439

Table 3.11: Observed and Predicted l_x, Germany, Male, 1960-69

Age	1967-69		1964-66		1960-62	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
0	1	1	1	1	1	1
1	0.9736	0.97423	0.9728	0.97286	0.9657	0.96467
2	0.9719	0.97248	0.9711	0.97108	0.9636	0.96244
5	0.9693	0.96983	0.9684	0.96828	0.9604	0.95929
10	0.9665	0.96678	0.9656	0.96532	0.9569	0.9562
15	0.9644	0.96442	0.9633	0.9629	0.9543	0.95388
20	0.9582	0.95794	0.9571	0.95689	0.9474	0.94812
25	0.9502	0.95029	0.9490	0.94902	0.9386	0.93948
30	0.9431	0.94313	0.9418	0.94169	0.9307	0.93166
35	0.9350	0.93496	0.93364	0.93331	0.9220	0.92322
40	0.924	0.92366	0.9226	0.92223	0.9104	0.91218
45	0.9078	0.90723	0.9064	0.90617	0.8937	0.89659
50	0.8824	0.88145	0.8809	0.88069	0.8672	0.8723
55	0.8429	0.84173	0.8411	0.84209	0.8257	0.83221
60	0.7801	0.77808	0.7779	0.77735	0.75859	0.76652
65	0.6859	0.67929	0.6830	0.67895	0.6581	0.66941
70	0.5574	0.54191	0.5539	0.54801	0.5232	0.54461
75	0.4033	0.38355	0.3995	0.39745	0.3676	0.39784
80	0.2441	0.22889	0.2409	0.24324	0.2137	0.24156
85	0.1118	0.10278	0.1097	0.11243	0.0925	0.10861
90	0.03407	0.02987	0.03314	0.03461	0.02589	0.03092

Table 3.12: Observed and Predicted l_x, Germany, Female, 1982-96

Age	1994-96		1992-94		1982-84	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
0	1	1	1	1	1	1
1	0.9956	0.99533	0.9950	0.99486	0.9909	0.99094
2	0.9952	0.99488	0.9946	0.99433	0.9902	0.99024
5	0.9945	0.99422	0.9938	0.99363	0.9891	0.98917
10	0.9938	0.99358	0.9930	0.99293	0.9880	0.98811
15	0.9930	0.99295	0.9923	0.99222	0.98712	0.98725
20	0.9912	0.99137	0.9904	0.99064	0.9851	0.98519
25	0.9893	0.98965	0.9885	0.98892	0.9829	0.983
30	0.9873	0.98782	0.9864	0.98692	0.98045	0.98047
35	0.9844	0.98506	0.9835	0.98394	0.9771	0.97711
40	0.9799	0.98068	0.9789	0.97938	0.9719	0.97184
45	0.9728	0.97353	0.9716	0.97196	0.9638	0.96404
50	0.9615	0.96257	0.9601	0.96069	0.9512	0.95199
55	0.9445	0.9464	0.9430	0.94447	0.9324	0.93309
60	0.9197	0.92339	0.9178	0.92014	0.9048	0.90507
65	0.8817	0.8866	0.8792	0.88171	0.8622	0.86123
70	0.8234	0.82917	0.8198	0.82257	0.7957	0.79521
75	0.7289	0.73737	0.7237	0.72707	0.6893	0.68985
80	0.5838	0.59829	0.5768	0.58501	0.5311	0.52761
85	0.3857	0.40547	0.3781	0.38992	0.3301	0.32332
90	0.1448	0.19839	0.1424	0.18729	0.12749	0.13893

Table 3.13: Observed and Predicted l_x, Germany, Female,1972-81

Age	1979-81		1977-79		1972-74	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
0	1	1	1	1	1	1
1	0.9887	0.98897	0.9871	0.98736	0.9812	0.98104
2	0.9879	0.98806	0.9861	0.98642	0.9798	0.97992
5	0.9866	0.98674	0.9848	0.98501	0.978	0.97819
10	0.9854	0.9855	0.9834	0.98354	0.9763	0.97637
15	0.9844	0.98443	0.9824	0.98239	0.9750	0.9751
20	0.9823	0.98211	0.9802	0.97977	0.9725	0.97237
25	0.9799	0.97971	0.9778	0.977	0.9697	0.96952
30	0.9774	0.97685	0.9750	0.97396	0.9665	0.96644
35	0.9737	0.9731	0.9712	0.9699	0.9620	0.96214
40	0.9682	0.96754	0.9655	0.96423	0.9555	0.95589
45	0.9598	0.95927	0.9567	0.95544	0.9456	0.94595
50	0.9466	0.94586	0.9431	0.94117	0.9303	0.93042
55	0.9269	0.92575	0.9228	0.92021	0.9078	0.9068
60	0.898	0.89566	0.8930	0.88895	0.8747	0.87417
65	0.8534	0.85104	0.8468	0.84356	0.8231	0.82388
70	0.7834	0.78149	0.7742	0.77163	0.7414	0.74261
75	0.6719	0.66916	0.6591	0.65549	0.6141	0.61543
80	0.5086	0.50149	0.4923	0.48628	0.4366	0.43781
85	0.3074	0.29799	0.2913	0.28344	0.2392	0.23932
90	0.1203	0.12206	0.1151	0.11442	0.0979	0.088801

Table 3.14: Observed and Predicted l_x, Germany, Male, 1964-69

Age	1967-69		1964-66	
	Observed	Predicted	Observed	Predicted
0	1	1	1	1
1	0.9798	0.98023	0.9799	0.98023
2	0.9783	0.97877	0.9783	0.97877
5	0.9764	0.97671	0.9764	0.97671
10	0.9746	0.9747	0.9746	0.9747
15	0.9732	0.97329	0.9732	0.97329
20	0.9707	0.97067	0.9707	0.97067
25	0.9678	0.96781	0.9678	0.96781
30	0.9645	0.96451	0.9645	0.96451
35	0.9599	0.96	0.9599	0.96
40	0.9532	0.9531	0.9532	0.9531
45	0.9430	0.94254	0.943	0.94254
50	0.9274	0.92601	0.9274	0.92601
55	0.9043	0.90206	0.9043	0.90206
60	0.8705	0.86723	0.8705	0.86723
65	0.8177	0.81174	0.8177	0.81174
70	0.7339	0.72318	0.7339	0.72318
75	0.6040	0.58811	0.6040	0.58811
80	0.4245	0.40685	0.4245	0.40685
85	0.2283	0.21272	0.2283	0.21272
90	0.0944	0.07391	0.0944	0.07391

Table 3.15: Mean square error and R^2 Germany, India, Nepal

Germany	R^2	**Mean square error**
Male		
1994-96	0.998312045	0.000154135
1992-94	0.999308172	6.48482E-05
1982-84	0.999766601	2.42715E-05
1979-81	0.999508604	5.23658E-05
1977-79	0.999489071	5.50544E-05
1972-74	0.999871121	1.43117E-05
1967-69	0.999474281	5.96794E-05
1964-66	0.999961398	4.261E-06
1960-62	0.998531841	0.000159686
Female		
1994-96	0.996185647	0.000213195
1992-94	0.997722414	0.00013235
1982-84	0.999833974	1.1251E-05
1979-81	0.999861107	9.94637E-06
1977-79	0.9998698	9.58638E-06
1972-74	0.999933013	5.38626E-06
1967-69	0.999040393	8.1983E-05
1964-66	0.999794654	1.72859E-05
1960-62	0.999194939	7.00676E-05

method of extrapolation is mentioned in the following way.

$$y_t = c_1 - c_2.e^{c_3.t}$$

$$y_1 = c_1 - c_2.e^{c_3.t_1}$$

$$y_2 = c_1 - c_2.e^{c_3.t_2}$$

$$y_3 = c_1 - c_2.e^{c_3.t_3}$$

y_1 is value of y (parameter)corresponding to the year 1990/92.
y_2 is value of y (parameter) corresponding to the year 1992/94.
y_3 is value of y (parameter) corresponding to the year 1994/96.

$$y_1 - y_2 = c_2(e^{c_3.t_2} - e^{c_3.t_1})$$

$$y_2 - y_3 = c_2(e^{c_3.t_3} - e^{c_3.t_2})$$

here, $t_2 - t_1 = 2$ and $t_3 - t_2 = 2$ thus,

$$c_3 = \frac{LN(\frac{(y_2 - y_3)}{y_1 - y_2})}{2}$$

India	R^2	**Mean square error**
male 1991	0.99991228	2.06684E-06
male 1990	0.999917033	2.00551E-06
male 1988	0.999999992	2.01568E-10
male urban 1991	0.999702732	5.871E-06
male urban 1990	0.999794431	4.14364E-06
male urban 1988	0.999994121	1.37146E-07
male rural 1991	0.999904138	2.37343E-06
male rural 1990	0.999912473	2.21898E-06
male rural 1988	0.99999992	2 1.9562E-09
female1991	0.99986165	2.30331E-06
female 1990	0.99987101	2.21279E-06
female 1988	0.999999977	4.10839E-10
female urban 1991	0.999506787	5.67044E-06
female urban 1990	0.999927028	8.70739E-07
female urban 1988	0.999989233	1.46918E-07
female rural 1991	0.999822739	3.23373E-06
female rural 1990	0.999836122	3.09261E-06
female rural 1988	0.999999963	7.07703E-10

Nepal	R^2	**Mean square error**
male 1991	0.970940782	0.000734861
male 1986	0.979507997	0.001365271
male 1976	0.996060103	0.000257597
female 1991	0.994797144	0.000168356
female 1986	0.992760233	0.000337445
female 1976	0.999558567	3.50018E-05

$$c_2 = \frac{y_2 - y_1}{e^{c_3.t_2} - e^{c_3.t_1}}$$

$$c_1 = y_3 + c_2.e^{c_3.t_3}$$

In this way c_1, c_2 and c_3 could be estimated and further substituted to find the value of y_t.

3.9 Conclusion

The objective of this chapter was to formulate a model specially suited to Nepal's needs. As mentioned on page 2 (Introduction), faults and errors in the population censuses and vital records are typical features of a statistically underdeveloped country. Inspite of such problems, the attempt to model the mortality scene was quite successful. The low value of M.S.E and the valuz of R^2 very close to 1 reflected this fact. Tables 3.1-3.14 further asserted these facts. The model was quite successful in modeling the mortality scenes, of Nepal, India and Germany. It showed better results compared to the Denny's model and Lee and Carter's model mentioned in Chapter 2. Tables 2.3-2.6 further verified this fact.

Nepal As seen from Table 3.18, the life expectancy at birth for females rose from 34.3 in 1966 to 57.08 in 1998. The ${}_nL_x$ values, needed to calculate the expectation at birth, were obtained from ${}_nl_x$ values using Cubic approximation [55].

$${}_nL_x = \frac{13n(l_x + l_{x+n})}{24} - \frac{n(l_{x-n} + l_{x+2n})}{24}$$

The life expectancy of women of Nepal doubled in the last 50 years. In the case of males (Table 3.19), it increased from 41.9 in 1966 to 53.9 in 1998. Thus, the increase in life expectancy in the case of females was more than that of males. Table 2.1 was the data source for Nepal.

India: The model showed better results for India as compared to Nepal. Big age groups and data available after long time gaps were the resons behind the relatively poor performance of this model in case of Nepal. With the help of Cubic approximation again, the predicted (pre.) expected longivity at birth and Observed (obs.) expected longivity at birth (Table 3.16-3.17) were compared.

In the case of females in 1975 the e_0 (pre.) was 51.10 whereas e_0 (obs.) 52.1. During 1970 e_0 (pre.) was 50.7 whereas e_0 (obs.) 49.0. During the period 1980 - 85 e_0 (pre.) was 58.43 whereas e_0 (obs.) 58.1. During 1988-92 e_0 (pre.) was 59.05 whereas e_0 (obs.) 59. During 1991 - 95 e_0 (pre.) was 60.42 whereas e_0 (obs.) 60.9.

Similarly in the case of males in 1970-75 the e_0 (pre.) was 51.57 whereas e_0 (obs.) 50.5. During 1975-80 the e_0 (pre.) was 52.7 and e_0 (obs.) was 52.5. During the period 1985-90 e_0 (pre.) was 58.3 whereas e_0 (obs.) 57.7. During 1988-92 e_0 (pre.) was 58.6 whereas e_0 (obs.) 58.6. During 1991-95 e_0 (pre.) was 59.7 whereas e_0 (obs.) 59.7.

Thus the model could project the results for the previous years quite accurately, as shown by the very close correspondence between the observed and expected value.

Further the life expectancy of female rose from 46.6 years during the period 1945-50 to 66.2 in 2006. In the case of males, the life expectancy rose from 46.3 years in 1945 to 61.16 years in 2009.

The life expectancy data as old as 1946 were lacking. The age-specific data were more rare and inaccurate. So, with the help of this model, an assessment of the overall improvement in the life expectancy from 1946 till date for both Nepal and India was possible.

Germany: Contrary to Nepal and India, Germany has very accurate and abundant data. Additionally, it falls into the category of those countries, which had achieved a very high life expectancy as early as 1950's. So, here the *improvement* in the life expectancy was not so high as compared to Nepal and India. For females e_0 (pre.) 2000 is 79.6, e_0 (pre.) 2005 is 80.3, e_0 (pre.) 2008 is 80.6. For males e_0 (pre.) 2000 is 73.34, e_0 (pre.) 2003 is 73.6, e_0 (pre.) 2005 is 73.8 (Table 3.21-3.20).

It was also been observed that, short term prediction could be made for India and Germany with the help of this model. The reason behind it was the accurate and abundant data. In the case of Germany, the model was based on large set of l_x values, with a large set of time periods, which were lying close to one another, ranging from 1962 to 1994. The age-groups on which l_x was based were small. In the case of India, the age groups on which l_x was based were small, but it has a very small set of time period, which were lying very close to one- another. Thus, while projecting into the future, this model has functioned not so well for Nepal as compared to India and Germany.

Table 3.16: Projected l_x; 1945-2009; Male; India

Age	1945	1955	1965	1975	1985	1995
0	1	1	1	1	1	1
1	0.8650	0.8767	0.8885	0.9006	0.9128	0.9251
5	0.7775	0.8011	0.8254	0.8505	0.8763	0.903
10	0.7529	0.7794	0.8068	0.8352	0.8645	0.8949
15	0.7435	0.7707	0.7988	0.8280	0.8583	0.8897
20	0.7349	0.7623	0.7907	0.8201	0.8507	0.8824
25	0.7179	0.7467	0.7767	0.8078	0.84021	0.8740
30	0.7087	0.7372	0.7667	0.7974	0.8294	0.8627
35	0.6981	0.726	0.7550	0.7852	0.8166	0.8493
40	0.6771	0.7059	0.7359	0.7671	0.7997	0.8337
45	0.6464	0.6767	0.7085	0.7417	0.7765	0.813
50	0.6036	0.6354	0.6689	0.7042	0.7412	0.7803
55	0.5526	0.5846	0.6186	0.6545	0.6925	0.7327
60	0.4860	0.5173	0.5506	0.5861	0.6238	0.664
65	0.3886	0.4199	0.4536	0.4900	0.5294	0.5719
70	0.3066	0.3317	0.3589	0.3883	0.4201	0.4545

Age	1998	2000	2003	2005	2008	2009
0	1	1	1	1	1	1
1	0.9288	0.9313	0.9351	0.9376	0.9414	0.9427
5	0.9111	0.9166	0.9248	0.9304	0.9388	0.9416
10	0.9042	0.9105	0.92	0.9264	0.9360	0.9392
15	0.8993	0.9058	0.9156	0.9222	0.9322	0.9355
20	0.8921	0.8987	0.9086	0.9152	0.9253	0.9287
25	0.8842	0.8912	0.9018	0.9089	0.9197	0.9233
30	0.8729	0.8798	0.8902	0.8973	0.9079	0.9115
35	0.8593	0.8661	0.8763	0.8832	0.8937	0.8972
40	0.8442	0.8512	0.8619	0.8691	0.8801	0.8837
45	0.8242	0.8318	0.8433	0.8511	0.8629	0.8669
50	0.7924	0.8006	0.8130	0.8214	0.8342	0.8384
55	0.7452	0.7536	0.7665	0.7752	0.7884	0.7929
60	0.6765	0.6850	0.698	0.7067	0.7201	0.7246
65	0.5853	0.5944	0.6084	0.6178	0.6323	0.6372
70	0.4654	0.4728	0.4841	0.4918	0.5035	0.5075

Table 3.17: Projected l_x; 1945-2009; Female; India

Age	1945	1955	1965	1975	1985	1995
0	1	1	1	1	1	1
1	0.8744	0.8844	0.8946	0.9049	0.9153	0.9258
5	0.7378	0.7671	0.7975	0.8291	0.8619	0.8961
10	0.7096	0.7418	0.7756	0.8108	0.8476	0.8861
15	0.6998	0.7326	0.7670	0.8030	0.8407	0.8801
20	0.6803	0.7149	0.7512	0.7893	0.8294	0.8715
25	0.6731	0.7067	0.7421	0.7792	0.8181	0.859
30	0.6646	0.6975	0.732	0.7681	0.8061	0.8459
35	0.6584	0.6900	0.7232	0.75800	0.7944	0.8326
40	0.6409	0.6732	0.7072	0.7429	0.7803	0.8197
45	0.6210	0.6539	0.6885	0.7250	0.7635	0.8039
50	0.5921	0.6261	0.6622	0.7002	0.7405	0.7831
55	0.5548	0.5893	0.6258	0.6647	0.7059	0.7497
60	0.4901	0.5269	0.5664	0.6088	0.6544	0.7035
65	0.4183	0.4537	0.4921	0.5337	0.5789	0.6279
70	0.2928	0.3301	0.3723	0.4197	0.4733	0.5337

Age	1998	2000	2003	2006
0	1	1	1	1
1	0.92900	0.9311	0.9343	0.9375
5	0.9066	0.9137	0.9244	0.9352
10	0.8980	0.9060	0.9182	0.9305
15	0.8923	0.9006	0.9130	0.9257
20	0.8846	0.8933	0.9067	0.9203
25	0.8716	0.8802	0.8932	0.9063
30	0.8583	0.8666	0.8792	0.8920
35	0.8444	0.8524	0.8644	0.8767
40	0.8319	0.8401	0.8526	0.8653
45	0.8165	0.825	0.8378	0.8509
50	0.7964	0.8053	0.819	0.8328
55	0.7634	0.7726	0.7867	0.8010
60	0.7189	0.7294	0.7453	0.7617
65	0.6434	0.6539	0.6701	0.6866
70	0.5532	0.5667	0.5875	0.6090

Table 3.18: Projected l_x; 1945-2001; Female, Nepal

Age	1946	1956	1966	1976	1996	1997	1998	2001
0	1	1	1	1	1	1	1	1
1	0.8427	0.8576	0.8728	0.8882	0.9199	0.9215	0.9232	0.9280
5	0.6345	0.6762	0.7208	0.7682	0.8728	0.8783	0.8839	0.9010
15	0.5648	0.6120	0.6632	0.7186	0.8437	0.8505	0.8573	0.8782
25	0.5145	0.5641	0.6183	0.6778	0.8146	0.8221	0.8297	0.8529
35	0.4183	0.4752	0.5398	0.6132	0.7914	0.8016	0.8118	0.8435
45	0.3106	0.3716	0.4446	0.5320	0.7616	0.7754	0.7895	0.8331
55	0.1338	0.1878	0.2637	0.3702	0.7297	0.7549	0.7810	0.8647
65+	0.0475	0.0799	0.1344	0.2260	0.6393	0.6735	0.7094	0.8292

Table 3.19: Projected l_x; 1946-2001; Male; Nepal

Age	1946	1956	1966	1976	1996	1997	1998	2001
0	1	1	1	1	1	1	1	1
1	0.8560	0.8681	0.8804	0.9183	0.9183	0.9195	0.9209	0.9235
5	0.6540	0.6969	0.7425	0.8983	0.8983	0.9040	0.9098	0.9214
15	0.7350	0.7529	0.7712	0.8288	0.8288	0.8308	0.8328	0.8368
25	0.7505	0.7598	0.7692	0.7981	0.7981	0.7991	0.8001	0.8021
35	0.6610	0.6812	0.7020	0.7684	0.7684	0.7707	0.7730	0.7777
45	0.5844	0.6106	0.6381	0.7280	0.7280	0.7312	0.7344	0.7409
55	0.1696	0.2233	0.2939	0.6707	0.6707	0.6894	0.7086	0.7487
65+	0.0849	0.1238	0.1805	0.5601	0.5601	0.5816	0.6040	0.6513

Table 3.20: Projected l_x Germany, 2000-2008(female)

Age	2000	2005	bf 2008
0	1	1	1
1	0.997451948	0.998964225	0.999872052
2	0.997134539	0.998742898	0.999708479
5	0.996587869	0.998318178	0.999357074
10	0.995960942	0.997794364	0.998895262
15	0.99531251	0.997205835	0.998342757
20	0.993554982	0.995512547	0.996688108
25	0.99181285	0.99387491	0.995113331
30	0.989895696	0.992075224	0.99338432
35	0.987206816	0.989555354	0.990966163
40	0.98299558	0.98556669	0.987111494
45	0.976191874	0.979059043	0.98078217
50	0.965396351	0.968689384	0.970669197
55	0.949203426	0.953107961	0.955456731
60	0.925479371	0.930295126	0.933194552
65	0.889258714	0.895604034	0.899430238
70	0.834221552	0.843330219	0.848839195
75	0.744548109	0.757861494	0.765957789
80	0.605297477	0.623772993	0.635119833
85	0.409295001	0.430043771	0.442985309
90	0.151977523	0.15824085	0.162119309

Table 3.21: Projected l_x Germany, 2000-2005(male)

Age	2000	2003	2005
0	1	1	1
1	0.997529984	0.998797734	0.999618159
2	0.997217926	0.998558664	0.999426376
5	0.996572975	0.998018715	0.998954466
10	0.995836262	0.997392749	0.998400273
15	0.99499621	0.996625451	0.997680134
20	0.990919582	0.992663006	0.993791718
25	0.985807672	0.987705815	0.988934861
30	0.98101454	0.983041176	0.984353573
35	0.974803781	0.976934025	0.978313651
40	0.965593702	0.967821736	0.969264838
45	0.951293659	0.953625968	0.955136789
50	0.929444882	0.931972428	0.933610081
55	0.896338022	0.899226281	0.901098418
60	0.847978294	0.851688044	0.854094936
65	0.775735948	0.780749184	0.784007343
70	0.673640801	0.680345732	0.684715776
75	0.531951855	0.539713577	0.544791504
80	0.363402769	0.371044704	0.376070337
85	0.196958983	0.202878805	0.206800181
90	0.077387359	0.080780789	0.083053346

Chapter 4

Bayesian Analysis

4.1 Introduction

There was some kind of uncertainty over the data use, which was much higher in the case of Nepal as compared to Germany. Bayesian demography compels the demographer to face this uncertainty and express it in mathematical form. An effort was made, to model the uncertainty in the survivorship data of Nepal. A comparison was established between Germany (with accurate and abundant data) and Nepal (with inaccurate and scarce data). Each parameter of the model was looked at from the angle of uncertainty in the data. The parameter has been assigned a probabilistic distribution with mean and variance based on demographer's educated guess [15]. The basic idea behind the posterior and prior distribution is the assignment of a numerical value to the uncertainty. Thus, the use of the word Bayesian Demography is justified.

4.2 Application

It was shown before that the model $ln(\frac{1}{l_x}) = (a_x+b_x.k_t)(\frac{x}{105-x})$ had given very good results with very good values of R^2 and MSE. The fit was very good for Germany with R^2 as high as 0.999 and for Nepal 0.9768. The model has shown better results for Germany (very accurate data) and India (country with reasonably accurate data) than Nepal. Here the M.A.D (Mean absolute deviation) between observed and expected l_x was used to guess the value of s (standard deviation). In the case of information on the uncertainity of the data, this information could have been utilized to estimate s.

The uncertainty in k_t, a_x and b_x was modeled in the following way.

For given t and x, k_t, a_x and b_x are normally distributed.

$$k_t \sim N(k'_t, s^2)$$

$E(k_t) = k'_t$, where k'_t is the calculated value of k_t (most frequently observed value)
s expresses the scatterness in k_t given t and thus the uncertainty. The higher the value of s the more uncertain is the data. Similarly,

$$a_x \sim N(a'_x, s_1^2)$$

$$b_x \sim N(b'_x, s_2^2)$$

Where a'_x, b'_x are the calculated values of a_x, b_x.

Here s was estimated by M.A.D (Mean absolute deviation) for the year t.

Since a_x, b_x were age dependent, and the data on young ages and very old ages (Nepal) were nor so reliable, some individual age adjustments have been made in s_1, s_2.

s_1, s_2 were taken as maximum M.A.D taking all the years into consideration.

s expressed uncertainty, thus s for Germany for the same time t was much lower as compared to s for Nepal, taking for example k_t into consideration.

Thus for given t,

$$P[E(k_t) - pE(k_t) < k_t < E(k_t) + pE(k_t)] = 1 - \alpha$$

for given x,

$$P[E(a_x) - pE(a_x) < a_x < E(a_x) + pE(a_x)] = 1 - \alpha$$

$$P[E(b_x) - pE(b_x) < b_x < E(b_x) + pE(b_x)] = 1 - \alpha$$

when constructing 95 percent confidence interval

$$P[E(k_t) - pE(k_t) < k_t < E(k_t) + pE(k_t)] = .95$$

now p has to be estimated; $z = \frac{k_t - E(k_t)}{s}$

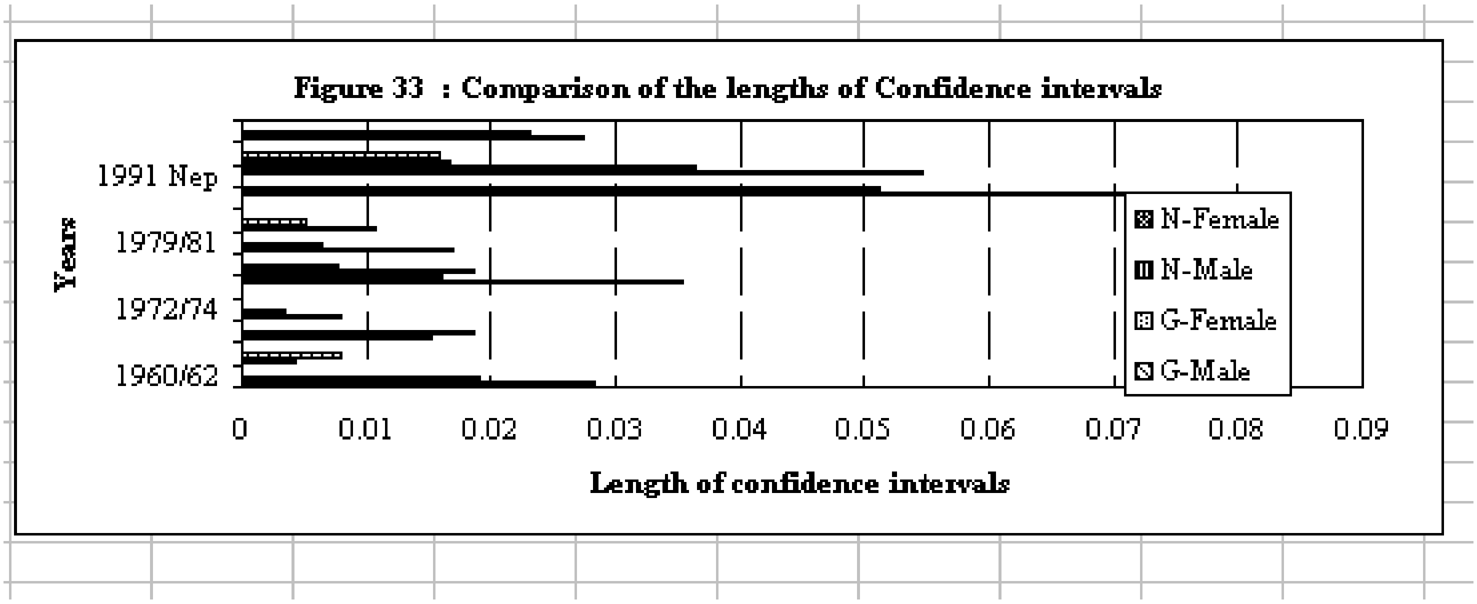
$$P[\frac{-pE(k_t)}{s} < z < \frac{pE(k_t)}{s}] = .95$$

Using the standard normal tables.

$$\frac{pE(k_t)}{s} = 1.96$$

$$p = \frac{1.96s}{k'_t}$$

similarly 95 percent confidence interval can be constructed for a_x and b_x.



4.3 Interpretation of the result

As seen from Figure 33, the length of the confidence interval was much smaller in the case of Germany than in the case of Nepal (Table 4.1-4.5). The length of the confidence interval shows the degree of concentration around the expected values, which further expresses the degree of certainty or uncertainty associated with the parameter values. Further for Nepal, the confidence interval is longer for males as compared to females. The faulty data on male population is perhaps the reason behind it. The data on males are subjected to more errors as compared to females. More interra-country occupational migration by males as compared to females and reluctance to report the death of a male child or a male family members are few of many reasons.

Table 4.1: k_t: 95 percent Confidence Interval

Germany age-group	$E(k(t))$ male	s	p	L.C.I	U.C.I
1960/62	4.1641	0.007219	0.003398	4.1499	4.1783
1964/66	2.2672	0.001119	0.000968	2.26500	2.2694
1967/69	2.0534	0.0039	0.00370	2.0458	2.0610
1972/74	1.7042	0.00204	0.0024	1.700154	1.7082
1977/79	-0.4323	0.00473	-0.0215	-0.4416	-0.4231
1979/81	-1.0382	0.004298	-0.00812	-1.046628	-1.0298
1982/84	-1.852	0.002717	-0.002876	-1.8573	-1.8467
1992/94	-3.3162	0.004244	-0.0025	-3.3245	-3.3079
1994/96	-3.5501	0.00701	-0.003869	-3.5639	-3.5364

Germany age-group	$E(k(t))$ female	s	p	L.C.I	U.C.I
1960/62	-2.6985	0.0049	-0.003529	-2.7080	-2.6890
1964/66	-2.5386	0.00203	-0.0016	-2.5426	-2.5346
1967/69	-1.44100	0.00476	-0.006472	-1.4503	-1.4317
1972/74	-0.8668	0.00087	-0.00197	-0.8685	-0.8650
1977/79	-0.4326	0.00198	-0.00896	-0.4365	-0.42875
1979/81	1.1609	0.00165	0.0028	1.1576	1.1641
1982/84	1.5348	0.00132	0.00168	1.5322	1.5374
1992/94	1.83896	0.004048	0.004315	1.8310	1.84689
1994/96	3.4429	0.0059	0.00336	3.4313	3.4544

Nepal age-group	$E(k(t))$ male	s	p	L.C.I	U.C.I
1976	2.8291	0.00899	0.00623	2.8114	2.8467
1986	-0.0069	0.02022	-5.7745	-0.0465	0.0328
1991	-2.8222	0.01393	-0.0097	-2.8495	-2.7949

Nepal age-group	$E(k(t))$ female	s	p	L.C.I	U.C.I
1976	4.1433	0.00408	0.00193	4.1353	4.1513
1986	-0.7683	0.013078	-0.0334	-0.7939	-0.7427
1991	-3.3750	0.0093	-0.0054	-3.3932	-3.3568

Table 4.2: a_x, male, 95 percent Confidence Interval

Germany age-group	$E(a(x))$ male	s	p	L.C.I	U.C.I
1	1.959	0.0072	0.0072	1.9450	1.9733
2	1.0350	0.0072	0.01367	1.0209	1.0492
5	0.4422	0.0072	0.032	0.4281	0.4564
10	0.2308	0.0072	0.0613	0.2167	0.2449
15	0.1566	0.0072	0.0904	0.1424	0.1707
20	0.1350	0.0072	0.1048	0.1209	0.1492
25	0.1251	0.0072	0.1132	0.1109	0.1392
30	0.1144	0.0072	0.1237	0.1002	0.1285
35	0.1073	0.0072	0.1319	0.0931	0.1214
40	0.1052	0.0072	0.1346	0.0910	0.1193
45	0.1086	0.0072	0.1302	0.0945	0.1228
50	0.1189	0.0072	0.1189	0.1048	0.1331
55	0.1372	0.0072	0.1032	0.1230	0.1513
60	0.1659	0.0072	0.0853	0.1517	0.1800
65	0.2073	0.0072	0.0682	0.1932	0.2215
70	0.2613	0.0072	0.0541	0.2472	0.2755
75	0.3271	0.00721	0.0433	0.3129	0.3413
80	0.4001	0.0072	0.0354	0.386	0.4143
85	0.4721	0.0072	0.0299	0.4579	0.4862
90	0.5186	0.0072	0.0273	0.5045	0.5328

Nepal age-group	$E(a(x))$ male	s	p	L.C.I	U.C.I
1	10.5208	0.0400	0.0075	10.4424	10.5992
5	3.5836	0.0202	0.0111	3.5440	3.6233
15	1.2899	0.0202	0.0307	1.2503	1.3295
25	0.7662	0.0202	0.0517	0.7265	0.8058
35	0.5951	0.0202	0.0666	0.5555	0.6348
45	0.4897	0.0202	0.0809	0.4501	0.5294
55	0.6464	0.0300	0.0910	0.5876	0.7052
65	0.6200	0.1000	0.3162	0.4240	0.8160

Table 4.3: b_x, male 95 percent Confidence Interval

Germany age-group	$E(b(x))$ male	s	p	L.C.I	U.C.I
1	0.4007	0.007219	0.0353	0.3866	0.4149
2	0.2099	0.0072	0.0674	0.1958	0.2241
5	0.0879	0.0072	0.1609	0.0738	0.1021
10	0.0450	0.0072	0.3144	0.0309	0.0591
15	0.0298	0.00722	0.4751	0.0156	0.0439
20	0.0227	0.0072	0.6243	0.0085	0.0368
25	0.0187	0.0072	0.7577	0.00453	0.0328
30	0.0157	0.0072	0.9039	0.0015	0.0298
35	0.0132	0.00722	1.0683	-0.00090	0.0274
40	0.0114	0.00722	1.2453	-0.00279	0.0255
45	0.0099	0.0072	1.4285	-0.004244	0.0241
50	0.009063	0.007219	1.5613	-0.0051	0.0232
55	0.0089	0.0072	1.5947	-0.0053	0.0230
60	0.00993	0.00722	1.4246	-0.0042	0.02408
65	0.01203	0.0072	1.1765	-0.0021	0.0262
70	0.0150	0.007219	0.9418	0.0008738	0.0292
75	0.017579	0.007219	0.8049	0.00343	0.0317
80	0.01973	0.00722	0.71722	0.00558	0.0339
85	0.0211	0.0072	0.6694	0.00699	0.0353
90	0.0217	0.0072	0.6521	0.00755	0.03585

Nepal age-group	$E(b(x))$ male	s	p	L.C.I	U.C.I
1	0.4155	0.04	0.1887	0.3371	0.4939
5	0.3612	0.0202	0.1097	0.3216	0.4008
15	0.0410	0.0202	0.9660	0.0014	0.0807
25	0.0112	0.0202	3.5386	-0.0284	0.0508
35	0.0171	0.0202	2.3121	-0.0225	0.0568
45	0.01667	0.0202	2.3777	-0.0229	0.0563
55	0.0711	0.03	0.8267	0.0123	0.1299
65	0.0661	0.1	2.9659	-0.1299	0.2621

Table 4.4: a_x, female, 95 percent Confidence Interval

Germany age-group	$E(a(x))$ female	s	p	L.C.I	U.C.I
1	1.5193	0.0059	0.0076	1.5077	1.5309
2	0.8084	0.0059	0.0143	0.7968	0.8199
5	0.3445	0.0059	0.0336	0.3329	0.3561
10	0.1775	0.0059	0.0652	0.1659	0.1891
15	0.1189	0.0059	0.0973	0.1074	0.1305
20	0.0941	0.0059	0.1230	0.0825	0.1056
25	0.0792	0.0059	0.1461	0.0676	0.0908
30	0.0691	0.0059	0.1673	0.0576	0.0807
35	0.0636	0.0059	0.1820	0.0520	0.0751
40	0.0617	0.0059	0.1877	0.0501	0.0732
45	0.0633	0.0059	0.1829	0.0517	0.0748
50	0.0686	0.0059	0.1688	0.0570	0.0801
55	0.0771	0.0059	0.1501	0.0655	0.0887
60	0.0891	0.0059	0.1299	0.0775	0.1006
65	0.1070	0.0059	0.1081	0.0955	0.1186
70	0.1338	0.0059	0.0865	0.1223	0.1454
75	0.1744	0.0059	0.0663	0.1628	0.1860
80	0.2317	0.0059	0.0499	0.2201	0.2432
85	0.3028	0.0059	0.0382	0.2912	0.3144
90	0.3676	0.0059	0.0315	0.3560	0.3791

Nepal age-group	$E(a(x))$ female	s	p	L.C.I	U.C.I
1	10.8085	0.0500	0.0091	10.7105	10.9065
5	4.2102	0.0131	0.0061	4.1845	4.2358
15	1.5816	0.0131	0.0162	1.5560	1.6073
25	0.9993	0.0131	0.0257	0.9736	1.0249
35	0.7654	0.0131	0.0335	0.7398	0.7911
45	0.6422	0.0131	0.0399	0.6165	0.6678
55	0.6463	0.0700	0.2123	0.5091	0.7835
65	0.6486	0.0800	0.2418	0.4918	0.8054

Table 4.5: b_x, female, 95 percent Confidence Interval

Germany age-group	$E(b(x))$ female	s	p	L.C.I	U.C.I
1	0.3946	0.0059	0.0293	0.3830	0.4061
2	0.2079	0.0059	0.0557	0.1963	0.2194
5	0.0869	0.0059	0.1332	0.0753	0.0985
10	0.0438	0.0059	0.2645	0.0322	0.0553
15	0.0286	0.0059	0.4052	0.0170	0.0401
20	0.0209	0.0059	0.5524	0.0094	0.0325
25	0.0166	0.0059	0.6952	0.0051	0.0282
30	0.0138	0.0059	0.8404	0.0022	0.0253
35	0.0119	0.0059	0.9723	0.0003	0.0235
40	0.0106	0.0059	1.0885	-0.0009	0.0222
45	0.0098	0.0059	1.1816	-0.0018	0.0214
50	0.0094	0.0059	1.2336	-0.0022	0.0210
55	0.0093	0.0059	1.2382	-0.0022	0.0209
60	0.0097	0.0059	1.1871	-0.0018	0.0213
65	0.0110	0.0059	1.0560	-0.0006	0.0225
70	0.0136	0.0059	0.8510	0.0020	0.0252
75	0.0178	0.0059	0.6518	0.0062	0.0293
80	0.0235	0.0059	0.4918	0.0120	0.0351
85	0.0291	0.0059	0.3971	0.0176	0.0407
90	0.0169	0.0059	0.6865	0.0053	0.0284

Nepal age-group	$E(b(x))$ female	s	p	L.C.I	U.C.I
1	0.3650	0.0500	0.2685	0.2670	0.4630
5	0.2552	0.0131	0.1004	0.2296	0.2809
15	0.0963	0.0131	0.2660	0.0707	0.1220
25	0.0588	0.0131	0.4357	0.0332	0.0845
35	0.0510	0.0131	0.5023	0.0254	0.0767
45	0.0479	0.0131	0.5355	0.0222	0.0735
55	0.0617	0.0700	2.2228	-0.0755	0.1989
65	0.0640	0.0800	2.4492	-0.0928	0.2208

Chapter 5

Introduction of Fertility

Fertility is of interest to demographers and in some cases more interesting than mortality. The reason for this preference is that mortality as an event occurs only once in a person's life time. Fertility on the other hand occurs more frequently.

The word fecundity is the biological capacity to reproduce [34], which varies from woman to woman, and from age to age. It can be studied either as a function of marriage or independently. The stigma associated with illegitimacy varies from place to place and from time to time, so the pressure on a couple to marry in order to have children or to legitimize a conception out of wedlock will vary correspondingly. In Nepal, for example, the social customs and traditions prevent a couple to have children outside a wedlock.

In many developing countries a history of accurately recorded fertility data is strikingly missing. Reliable responses about illegitimate infants cannot be expected. In addition to that, when asked about the number of children, the information about deceased children, especially girls, are not always remembered. In typical enquiries the ratio of girls to boys recorded falls with increasing length of time married, as illustrated by following representative figure below.

Married for less than $5-$years ratio .95

Married for $10-14$ years ratio .90

Married for $20-24$ years ratio .85

In some African countries an understatement of fertility by 25 percent is normal.

Since the vital rates in India was lacking [18], the level of fertility in that country had to be estimated from the census data or from data obtained from the National Sample Survey and the Sample Registration System of the Registrar General of India. Various scholars had tried to estimate and study the level and trends of fertility in India.

The crude birth rate for example for the intercensal decade 1951- 1961 was estimated at 41.7 per 1000. Similarly, on the basis of data obtained from the Sample registration system, an estimated Crude birth rate of 37 for 1970 was derived. An important feature of the Indian demographic situation was the very early marriage of women. It had been suggested, on the basis of 1960-1970 survey data, that the average age at marriage of women, though still comparatively low, had risen significantly from 13 years prior to 1951 to 18 years in 1966-70.

Only highly tentative estimates were available for Nepal. A variety of estimates were made of the level of Nepalese fertility, using different analytical methods. These were based on the data obtained from the censuses of 1952-54 and 1961.

In 1974, a world fertility (WFS) survey was begun by the International Statistical Institute in conjunction with United Nations. Importance of collecting good quality data and determinants of fertility was emphasized.

In contrast to developing countries, the developed countries have a long tradition of accurately recorded demographic data.

Measures:
There are various measures of Fertility, such as Crude Birth Rate (CBR), Total Fertility Rate among ever married women (TFR), Age Specific Fertility rates (ASFR) and the mean number of children ever born. TFR is defined as the sum of all age specific fertility rates and is interpreted as the total number of children a woman would bear by the end of her reproductive period if the prevailing fertility rates were to apply.

ERRORS:
As mentioned in [21], age mis-reporting, recall problems and problems in dating of events are some of the errors involving the data. These errors can affect not only the estimation of trends in fertility but also recent fertility estimates. To evaluate the quality of data during WFS, internal and external checks of the data with respect to women's age, their union status at the time of the survey, their birth histories were made. All these were important inputs used in the calculation of various measures of fertility.

Age reporting for example, was examined for digit preference, completeness of reporting of birth dates and distortions in five-year female age distribution. Household data and data from an external source, mostly from recent census source were compared to check the female age distribution data.

By studying the completeness of reporting of dates of marriage, the quality of the marriage history was assessed. In addition, the single year distribution of marriage duration was examined for suggestions of preference for particular duration, and trends in proportions married were compared with marital status distributions from other censuses and surveys.

Total Fertility Rate (TFR):
According to the World Fertility Survey (1974) conducted in 38 countries[30], the TFR ranged from four children per women in Costa Rica, Sri Lanka, Trinidad and Tobago to more than eight children in Kenya and Yemen. Nepal stood in between with a TFR of six children per women.

Global levels and trends:
As mentioned in [22], on observing the trend of TFR for the decade 1980-85 to 1990-95, it could be concluded that TFR in the world continued to decline. The pace of decline was faster in the recent five years than in the first half of the decade. Continuing fertility transition in most of the less developed regions had combined with a recent onset of fertility decline in sub-Saharan Africa and a sudden drop of birth rates in Eastern Europe. During the decade, the world TFR fell by 17 percent, from 3.6 to 3.0 births per woman. Among larger countries (with population more than 20 million in 1995) the overall 20 - year drop in fertility levels varied from a decline of fewer than 1.5 births per woman in the Islamic Republic of Iran, Iraq, Pakistan and Philippines to 3.0 or more in Algeria, Bangladesh, China, Mexico, Morocco and Thailand, while smaller countries were characterized by even larger variations.

The fastest drop was recorded in the 1970s and the early 1980s in the less developed regions. The TFR dropped from 4.9 in 1965-70 to 3.0 in 1990-95. The difference between more developed and the less developed regions more than halved from 3.3 births per woman in 1970-75 to 1.6 births in 1990-95.

There is a strong interrelationship between fertility and mortality. As early as 1961, Ronald Freedman wrote that a decline in mortality must produce a decline in fertility. Historically, data from several European countries show a certain relationship between fertility and mortality [32]. The reason for the explanation of fertility through mortality are given by Mahadevan [32]as the following.

1. The child mortality results in curtailed lactation period. This results in increased fecundity and shortened intervals between two consecutive births.

2. The mortality of a child motivates the parents to replace this loss through more children.

3. Because of high child mortality, the parents want the number of the children to be big enough, so that a sufficient number survives till the adulthood.

The poor people, suffering from nutritional deficiency have large number of children. High fertility together with lack of adequate food, causes deficiency resulting in mortality. Hence fertility and mortality are also linked through food availability and distribution. In Lutz [37], defined fertility in a new way, in terms of its demographic dimensions.

1. **The age of the mother** is one of the basic dimensions of human fertility. The fertility of a woman is directly dependent on her biological age. Menarche (the onset of first menstruation) and menopause set bounds to the reproductive age span. Within this age span births are unevenly distributed.

2. **Order of birth, parity**: The fertility shows dependence on parity. There is a strong correlation between age (declining fecundability) and parity. The reason behind it is that, with an increase in parity, there is an increased likelihood of sterility. If reproductive behavior is within the calculus of conscious choice and is controlled in the way that the birth of an additional child depends on the number of children already born, then the order of birth is the most important demographic dimension with reproductive age span.

3. **Duration of marriage:** In almost every society marriage is the major institution in which children are born and socialized. At every age the fertility of a married woman is many times that of a single, divorced or widow woman. Marital status is also an important discrete demographic dimension. **Time since the current marital status** is another important dimension of fertility.

4. **Duration since last birth** It is another factor which plays an important part as dimension of fertility. In contrast to duration of marriage which has neither physiological nor behavioral effect, both the factors affect duration since last marriage considerably.

5.1 Objective

The objective of the study was to present and to evaluate various models, which best describe Nepal's fertility conditions. The effort was concentrated in modeling the unimodal bell shaped Age Specific Fertility Rate curve with minimum number of parameters.

An effort was made towards the construction of a parsimonious model specially suited to Nepal.

While achieving this goal, steps were taken in the direction of identification and correction of errors in the data. Missing data have been taken care of through statistical procedures. A great effort was been put on the development of models with minimum number of parameters. There were some very good models, which gave not only very good results but also had large number of parameters. These models was very successful in predicting the mortality and fertility of countries, with an abundance of accurate data.

A comparison of fertility between various countries of South-Asia, for example India, Nepal was made.

5.2 Numerical methods

In an effort to find a suitable model, an extensive use of numerous computer software packages was made. For extensive numerical interpolation and extrapolation, EXCEL worksheets were very useful. In addition to that, EXCEL was intensively used for data mining and therefore, in the correction of faulty data. A rigorous use of EXCEL in the production of tables as well as in the construction of graphs was made. MATHEMATICA the computer algebra system has been used to find the fast solution of Symbolic mathematical problems (Such as pseudo inverse, higher order determinants, initial solution of non linear equations). Excel has not been programmed for Symbolic Mathematics. Mathematica has given quick and efficient solutions of such problems.

NAG FORTRAN library system and SPSS advanced statistical system were implemented in linear and non-linear optimization problems and modeling. In some cases they have complemented each other.

5.3 Data Source

For Nepal, Population Monograph of Nepal[11], was the main source of Fertility data and for India [25]. These publications have been collected from Nepal and India respectively. United nations publication, Demographic Yearbook have also been referred to for the data on fertility for countries like Bangladesh, India, Srilanka.

5.4 General Overview of Fertility Laws

Coale-Trussel[13] fertility law has achieved a pre-eminence among various fertility laws.

The reason behind it is, the explanation of the parameters of the law by social and biological processes governing fertility. That is, the parameters have not only mathematical meanings, but also social and biological meanings. Coale-Trussell model, a combination of nuptiality model and marital fertility modelis based on Coale-McNeil nuptiality model[14].

1. The Coale-McNeil nuptiality model is the model of frequency of first marriage amongst females. The model was first fitted to first marriage frequencies recorded in Sweden from 1865 to 1869[14]. Taking first marriage [46], only helps avoid the mathematical complications of divorce, widowhood and remarriage.

   $$r(a) = \frac{0.174}{k}.e^{-4.411.e^{-\frac{.309}{k}.(a-a_0)}} \tag{5.1}$$

   In this model, a represents the age.
   $r(a)$ is the risk of first marriage at age a ;
   a_0 is the earliest age at which there are a significant number of first marriages;
   k is the parameter which alters the shape of the curve;
   C is the parameter which scales the model. It is multiplied on the R.H.S of (5.1). It has no effect on the shape of (5.1)

   In developed countries $a_0 = 18$, the minimum age at marriage. k ranges from 0.2 (rapid marriage age after a_0) to 1.7 (very slow marriage rate). As mentioned before, the model was first fitted to Swedish women $1865-1869$. So, Swedish women $1865-69$ is the standard. In the case of a standard population $k = 1$, a value of 0.5 indicates that the rate of marriage is double than that in standard.

   Coale and McNeil (1972)[14]have tried to explain mathematically the entire marriage process. They explained every stage before marriage by a probability distribution.

   - The event arrival at the age of marriage is normally distributed.
   - The event of first meeting one's future husband is exponentially distributed.
   - The event of getting engaged is also exponentially distributed.
   - The event of finally getting married is also exponentially distributed.

2. The Coale Trussell Fertility models:

Table 5.1: Five-year values of $n(a)$ and $v(a)$ for Coale -Trussell fertility model.

Age-Group	$n(a)$	$v(a)$
20-24	.460	.000
25-29	.431	-.316
30-34	.396	-.814
35-39	.321	-1.048
40-44	.167	-1.424
40-44	.167	-1.424
45-49	.024	-1.667

As mentioned before, one of the strong points of this model is the biological meaning of the parameters, a combination of nuptiality model and marital fertility model. Marital fertility model relies on natural fertility, which in turn varies from population to population, mainly because of varying patterns of marriage. If the fertility is controlled through artificial ways, their is a change in fertility schedule (especially at ages which have high fertility). The new shape depends on the degree of departure from the uncontrolled fertility.

So, the Coale-Trussell fertility model:

$$r(a) = n(a).e^{m.v(a)} \tag{5.2}$$

a is the age.
r(a) is the proportion of marital fertility occurring at age a.
n(a) is the fixed schedule of natural fertility.
v(a) is the fixed schedule, expressing departure from natural fertility.
The value of n(a) and v(a) can also be seen Table 5.1.

This marriage assumes the absence of fertility outside marriage, and also absence of dissolution of marriage before the end of childbearing span of ages.

The model has two parameters. m indicates the extent of fertility. No fertility control takes place, when $m = 0.0$ and $m = 2.0$, indicates a very strong control. The parameter M multiplied on the RHS of (5.2), helps get the achieved total marital fertility.

$n(a)$, the average of natural fertility schedules collected by Henry.
$v(a)$, average of forty-three schedules of marital fertility found in 1965 United Nations Demographic Yearbook, calculated by Coale and Trussell. The various values of $n(a)$, $v(a)$(Table1) given in Coale-Trussell 1974 page 188 [13]

The combined model can be expressed algebraically as:

$$f(a) = G(a).n(a).e^{m.v(a)}$$

Where $f(a)$ is the age specific rate at age a.
$G(a)$ is computed from a_0 and k. It is the proportion ever married where the first marriage occurs according to a schedule characterized by the selected values of the parameter a_0 and k (equation (5.1)).

The choice of n(a) and v(a): The functions n(a) and v(a) are derived from empirical data. There were two steps in derivation: first the estimation of approximate values of n(a) and v(a) by five year age intervals above age 20, and second, determination of single year values by free hand interpolation above age 20 on somewhat arbitrary common sense principles.

Seven values of n(a) were derived by using the schedules designated by Henry [13].

Seven values of v(a) at ages 20-24 through 45-49, were obtained by calculations employing the marital fertility schedules listed in the United nations Demographic Year Book (United Nations 1966). Schedules known or suspected to be distorted by age mis-reporting or other forms of faulty data were discarded. The details about the calculation of n(a) and v(a) can be obtained from [13].

3. Gompertz Relational Fertility Models:

   This system is similar to logit relational system of model life tables. But there, logit transformation is applied to l_x, the proportion of people surviving till age x. The relationship between the observed and the standard population (of logits of l_x) is expressed in terms of a straight line.

   In Gompertz relational model, Gompertz transformation is applied to Age Specific Fertility rates(ASFR) expressed as proportions (ie divided by the total fertility rates) and cumulative [6].

   Gompertz transformation is defined below:

$$Y(x) = -ln[-ln(\frac{F(x)}{F})] \qquad (5.3)$$

   F(x) is cumulative ASFR upto age x.
   F is the total fertility.

It is assumed that $\frac{F(x)}{F} = A^{B^x}$

$$-ln[\frac{F(x)}{F}] = -B^x.ln(A)$$

$$ln(-ln(\frac{F(x)}{F})) = ln(-x.ln(B)) + ln[lnA]$$

So we get,

$$Y(x) = \alpha + \beta.Y_s(x)$$

where the standard is denoted by s. The standard fertility schedule represents fertility of high fertility countries. The parameters α and β have, as in the logit life table system, distinct interpretations. α changes the age location of the model. $\alpha = 0.0$ means the fertility pattern of observed population is the same as fertility pattern of standard population, indicating half of the total childbearing occurs by age 27(Table 1.2). $\alpha > 0.0$ indicates earlier fertility than in standard. β indicates spread of the schedule. Neutral value is 1.00.

The advantages of Gompertz model over Coale-Trussell model are the following.

(a) few parameters. It is a simple model with two parameters, in contrast to Coale-Trussell model with three parameters.

(b) Easy use, it does not require computers or long tables.

5.5 World Fertility Survey

High levels of fertility has lead to drastic population growth, specially in developed countries[30]. This was a problem of greatest worry, to the government and demographers. In addition to that, reliable information on birth rates was lacking and the information available was not trustworthy enough. Some statistics on current number of birth was available in South America for 51 percent population, in Africa for 73 percent population, and in Asia for 89 percent population.

Birth statistics by age of mother was available in South America for 49 percent population, in Africa for 19 percent population, and in Asia for 17 percent population.

The information by birth order was available in South America for 32 percent population, in Africa for 3 percent population, and in Asia for 10

Table 5.2: The standard fertility schedule

Age x	$-ln(-ln(F_s(x)))$ $Y_s(x)$	CumulFert $F_s(x)$	Age x	$-ln(-ln(F_s(x)))$ $Y_s(x)$	CumulFert $F_s(x)$
10	-	.00000	30	.70000	.60861
11	-3.17091	.00000	31	.84272	.65016
12	-2.742555	.00000	32	.99014	.68968
13	-2.36854	.00002	33	1.14407	.72722
14	-2.04079	.00045	34	1.30627	.76275
15	-1.75210	.00313	35	1.47872	.79618
16	-1.49286	.01168	36	1.66426	.82751
17	-1.25061	.03043	37	1.86597	.85663
18	-1.04479	.05826	38	2.08894	.88354
19	-.85927	.09428	39	2.33992	.90816
20	-.69130	.13584	40	2.62602	.93019
21	-.53325	.18187	41	2.95500	.94925
22	-.38524	.22993	42	3.32873	.96480
23	-.24423	.27897	43	3.75984	.97698
24	-.10783	.32829	44	4.25499	.98591
25	.02564	.37731	45	4.8097	.99188
26	.15853	.42597	46	5.41311	.99555
27	.29147	.47371	47	6.12864	.99782
28	.42515	.52013	48	7.07022	.99915
29	.56101	.56517	49	8.64839	.99982
30	.7000	.60861	50	$+\infty$	1.0000

percent population.

With regards to reliability, only 19 percent data in South America, 1 percent data in Africa and 8 percent data in Asia, were trustworthy.

These factors in late 1960's and early 1970's lead to the creation of World fertility survey. International Statistical Institute took the initiative to conduct this survey. The first survey was started in 1974. The WFS (world fertility survey) programme was completed as scheduled by middle 1984. **In Nepal, the year of field work was 1976**. WFS was funded by national and international agencies, including United nations fund for population activities.

Chapter 6

Some Fertility Models

6.1 The Double exponential model

Multiexponential models are single formulations, which can represent a wide variety of vital events (such as mortality, fertility, marriage). The age specific fertility rate ASFR, which is a unimodal bell shaped curve, can be represented by a four parameter (a, α, μ, β) double exponential curve.

$$m(x) = a.e^{-\alpha.(x-\mu)-e^{-\lambda.(x-\mu)}}$$

where a is the scaling parameter.
μ locates the schedule along x-axis and an increase in its value, while holding all the other parameters constant, moves the schedule to the right. α and λ are the slope parameters.

ASFR:

African countries show a high fertility rate in all age groups as compared to Latin America, Caribbean, Asia, Oceania[21]. This difference in the age specific fertility rates is prominent particularly in age groups 25-29 and 30-34, where the difference between the average for Africa and Latin America were 64 and 58 births per 1000 women, respectively. African ASFR are higher than that of Asian in all the age groups. When the strength of the family planning program and the level of development were considered, the countries with high development(group I) had lower fertility at all the age groups as compared to the countries with low development (group IV). The age specific rates for the groups II and III fell in between. Nepal, Pakistan, Bangladesh fall in group IV where as Sri Lanka is in group II.

ASFR with respect to the strength of the family planning program reflected the same trend. Countries with a strong family planning program had a lower fertility at all ages than those with a very weak or no program. Nepal, Bangladesh and Pakistan fall into the category of countries with weak efforts, whereas Sri Lanka falls into the category of moderate efforts.

6.1.1 The method of fitting

$$m(x) = a.e^{-\alpha.(x-\mu)-e^{-\lambda.(x-\mu)}}$$
$$= h(a, \alpha, \mu, \lambda, x)$$

Here x is the independent variable and it represents age.
a,α, μ, λ are the parameters to be estimated.

$$Q(a, \alpha, \mu, \lambda, x_i) = m(x) - h(a, \alpha, \mu, \lambda, x) \tag{6.1}$$

The Equation (6.1) represents the error e_i.
The objective is to choose $(a, \alpha, \mu, \lambda)$ from $\Re^4$ such that $\sum_i {e_i}^2$ would be minimal.

1. Simple Least Squares: The objective is to minimize $\sum_i {e_i}^2$. Here initial values of a, α, μ, λ is chosen. Through iterative scheme $\sum_i {e_i}^2$ is minimized. Levenberg-Marquardt method of non-linear optimization is used. That value of $(a, \alpha, \mu, \lambda, x)$ is chosen such that $\sum_i {e_i}^2$ is minimum.

2. Weighted Least Squares

   The objective is to minimize $\sum_i W_i {e_i}^2$, or minimise

   $$\sum_i W_i {e_i}^2 = \sum_i W_i (m(x_i) - h(a, \alpha, \mu, \lambda))^2 \tag{6.2}$$

   where W_i is the relative size of married women in the i-th age group.

   $$W_i = \frac{P_i}{P}$$

   where P_i is the number of married women in the i-th age group.
   P is the total number of married women.

   So it is reduced to minimize$\sum_i \frac{P_i}{P}(m(x_i) - h(a, \alpha, \mu, \lambda))^2$

   or, $\sum_i ((\frac{P_i}{P})^{\frac{1}{2}}.m(x_i) - (\frac{P_i}{P})^{\frac{1}{2}}.h(a, \alpha, \mu, \lambda))^2$

   Since social and religious customs hinder the births to be outside the wedlock, instead of taking women in reproductive age-group, married women in reproductive age span were taken. The Levenberg-Marquardt method of non-linear optimization was used.

Table 6.1: The age distribution married female

Age	**1971**	**1981**	**1991**	**1976** Interpolated	**1986** Interpolated
00-04	843600.66	1121107.959	1334897.712	990319.0028	1235967.529
05-09	855078.22	1069815.438	1371978.204	951518.6355	1209968.628
10-14	596833.12	791370.324	1121684.883	677129.5526	939555.4341
15-19	499273.86	630165.258	917742.177	545133.8689	754368.0274
20-24	505012.64	696112.785	862121.439	603699.1489	782253.5484
25-29	476318.74	593527.743	723069.594	533381.6355	656757.0625
30-34	424669.72	505597.707	602557.995	463129.6759	552073.8134
35-39	355804.36	432322.677	509856.765	393936.5471	470962.7496
40-44	309894.12	373702.653	435695.781	342025.3121	404926.1426
45-49	218073.64	285772.617	361534.797	250915.2281	322645.8066
50-54	195118.52	249135.102	287373.813	224099.0449	270226.6914
55-59	126253.16	161205.066	213212.829	141597.1309	185076.9654
60-64	154947.06	175860.072	213212.829	163348.5979	192481.4824
65+	183640.96	227152.593	315184.182	199831.782	265603.393
Total	5744518.78	7312847.994	9270123		

6.1.2 Interpolation

As seen from the table 6.1, the age distribution for the year 1976 and 1986 was interpolated using Polynomial Interpolation (Lagranges interpolation formula), which states that suppose population in 1971 is P_a and population in 1981 is P_b and population in 1991 is P_c, then we use three point interpolation to estimate the population of 1976. $Populationin1976 = f(1976)$

$$= P_a.\frac{(1976-1981).(1976-1991)}{(1971-1981).(1971-1991)} + P_b.\frac{(1976-1971).(1976-1991)}{(1981-1971)(1981-1991)}$$

$$+P_c.\frac{(1976-1971).(1976-1981)}{(1991-1971).(1991-1981)}$$

The Table 6.1 was obtained from the table percentage of currently married females [11]. This age-distribution table 6.1, obtained by the above mentioned method was used as weights in the weighted least square method.

6.2 The Gamma Density

While testing the efficacy of various models, which best reflect Nepal's fertility, a step was taken towards fitting a Gamma density.

The result was surprisingly good, better than the results obtained by other models. Hoem [29] has fitted Gamma density among several functions such as Coale-Trussell, Hadwiger, Beta to Danish fertility data, and his observation was the same. The performance of Gamma density was as good as Coale-Trussell, followed Hadwiger. Beta density did not show very promising results.

6.2.1 The Model

$$h(x; b, c, d) = \frac{1}{\Gamma b.c^b}(x-d)^{(b-1)}e^{\frac{-(x-d)}{c}} \tag{6.3}$$

Here d is the lowest childbearing age.
$\Gamma b = (b-1)!$
Let μ denotes the mean of the distribution.
Let m denotes the mode of the distribution.
σ^2 denotes the Variance.

6.2.2 The fitting

The model was fitted to the ASFR (Age specific fertility rate)data of Nepal. Let $h(.; \theta_1, \theta_2, \cdots, \theta_r)$ be the probability density function on a real line. $\theta_1, \theta_2, \cdots, \theta_r$ are the parameters whose optimum value has to be estimated. In this case $h(.; \theta_1, \theta_2, \cdots, \theta_r) = h(x; b, c, d)$, as three parameters in the probability density function has to be estimated. $0 < h(x; b, c, d) < 1$ where $x \epsilon \Re^+$. Here x denotes the age component.
Now,

$$g(x; R, \theta_1, \theta_2, \cdots, \theta_r) = R.h(x; \theta_1, \theta_2, \cdots, \theta_r) \tag{6.4}$$

R is the scaling parameter, R=TFR(Total fertility rate)
The ASFR data were fitted to (6.4).
The objective is to choose that value of $(R, \theta_1, \theta_2, \cdots, \theta_r)$ where

$$(R, \theta_1, \theta_2, \cdots, \theta_r) \epsilon \Re^{r+1}$$

such that

$$\sum_i (g(x_i; R, \theta_1, \theta_2, \cdots, \theta_r) - R.h(x; \theta_1, \theta_2, \cdots, \theta_r))^2$$

was minimized. The rates are available for age groups, rather than for single ages. Let m_j be the ASFR for the age group $(a_j, a_{j+1}]$, $j = 1, 2, \cdots, k-1$. Then

$$TFR = R = \sum_{j=1}^{k-1}(-a_j + a_{j+1}).m_j$$

$$M = Mean = \frac{1}{R}\sum_{j=1}^{k-1} x_j.m_j$$

Where x_j is the mid point of the age group $(a_j, a_{j+1}]$

$$S^2 = \frac{1}{R}\sum_{j=1}^{k-1} {x_j}^2.m_j - M^2$$

$$f(x) = (\frac{x}{b})^{(c-1)}.e^{\frac{-x}{b}}.\frac{1}{b.\Gamma c} \tag{6.5}$$

Where Γc is the gamma function of the parameter c.

$$\Gamma c = \int_0^{\infty} e^{-u}.u^{c-1}$$

Then $Mean = \mu = bc$
$Variance = \sigma^2 = b^2c$
$Mode = b(c-1)$ Comparing (6.4) and (6.5)

$$h(x) = \frac{1}{\Gamma b.c^b}(x-d)^{(b-1)}e^{\frac{-(x-d)}{c}}$$

let $x - d = y$
$dx = dy$
$h(y) = \frac{1}{\Gamma b.c^b}(y)^{(b-1)}.e^{-\frac{y}{c}}$ for $y > 0$

$$= \frac{1}{\Gamma b.c}(\frac{y}{c})^{(b-1)}.e^{-\frac{y}{c}}$$

where $y > 0$ Then

$$Mean = \mu = bc \tag{6.6}$$

$$Variance = \sigma^2 = c^2b \tag{6.7}$$

$$Mode = m = c(b-1) \tag{6.8}$$

From (6.6), (6.7) and (6.8) we get; $c = \mu - m$ $b = \frac{\mu-d}{c} = \frac{\sigma^2}{c^2}$. Substituting the sample estimates of μ, m, σ^2 the initial values of the parameters b, c, d can be estimated.

6.3 Hadwiger Function

Hoem [29] fitted the Hadwiger density to the Danish fertility data. The performance of Hadwiger density was second best, right next to Gamma and Coale-Trussell. Hadwiger density showed promising results, when fitted to the fertility data of Nepal.

6.3.1 The fitting

The model was fitted to the ASFR (Age specific fertility rate)data of Nepal. Let $h(.;\theta_1,\theta_2,\cdots,\theta_r)$ be the probability density function on a real line. $\theta_1,\theta_2,\cdots,\theta_r$ are the parameters whose optimum value has to be estimated. In this case $h(.;\theta_1,\theta_2,\cdots,\theta_r)=h(x;H,T,D)$, as three parameters in the probability density function has to be estimated. $0<h(x;H,T,D)<1$ where $x\epsilon\Re^+$. Here x denotes the age component.
Now,

$$g(x;R,\theta_1,\theta_2,\cdots,\theta_r)=R.h(x;\theta_1,\theta_2,\cdots,\theta_r) \tag{6.9}$$

R is the scaling parameter, R=TFR(Total fertility rate)
The ASFR data are fitted to (6.9).
The objective is to choose that value of $(R,\theta_1,\theta_2,\cdots,\theta_r)$ where

$$(R,\theta_1,\theta_2,\cdots,\theta_r)\epsilon\Re^{r+1}$$

such that

$$\sum_i\left(g(x_i;R,\theta_1,\theta_2,\cdots,\theta_r)-R.h(x;\theta_1,\theta_2,\cdots,\theta_r)\right)^2$$

was minimized.
The rates are available for age groups, rather than for single ages. let m_j be the ASFR for the age group $(a_j,a_{j+1}]$, $j=1,2,\cdots,k-1$. Then

$$TFR=R=\sum_{j=1}^{k-1}(-a_j+a_{j+1}).m_j$$

6.3.2 The model

$$h(x)=\frac{H}{T\sqrt{\Pi}}(\frac{T}{x-D})^{\frac{3}{2}}e^{-H^2(\frac{T}{x-D}+\frac{x-D}{T}-2)} \tag{6.10}$$

where $x>D$[29] The parameters $\theta_1=R$,

$$\theta_2=D+T\frac{(1+\frac{16.H^4}{9})^{\frac{1}{2}}-1}{\frac{4}{3}.H^2}$$

$$\theta_3=T+D$$

$$\theta_4=\frac{1}{2}.\frac{T^2}{H^2}$$

Where θ_2, θ_3, θ_4 are the mode, mean and variance respectively.

6.4 Gompertz Model

In earlier days, in order to smooth the mortality data, Gompertz formula was in frequent use. It may be written as,

$$\mu_x = Be^{kx}$$

where μ_x is the force of mortality of age at age x;
B and K are constants and
K lies in the vicinity of .09
This law had been used to describe the mortality experience of earlier times. It takes neither infant nor childhood mortality nor excess accident mortality in early adult life into consideration. Pollard[49] showed that using Gompertz law many short cuts and approximations (resulting from this law), help work out numerical problems faster. In addition to that, these formula are robust. Gompertz function has been quiet successful in describing fertility experience.

Cumulative Fertility:
Cumulative fertility (age specific) is the total number of children ever born to a woman belonging to a specific age group.
The women aged 40-49 in countries of high development and in countries with strong family planning program had about 6.1 children, whereas those in countries in a low development group and in countries with a very weak program had 5 children more at the end of the fertility span.

6.4.1 The Model

Here an attempt was made to fit a Gompertz function $Y = K.A^{B^t}$ to the cumulative fertility rates of Nepal. In addition to that, the mathematical meaning of the parameters of Gompertz function was analyzed.

$$Y = K.A^{B^t}$$

the origin is changed to t_0, so

$$Y(t) = K.A^{B^{t-t_0}} \tag{6.11}$$

Let $t_1 - t_0 = p$ $t_2 - t_0 = t_2 - t_1 + t_1 - t_0 = 2p$
the age points t_0, t_1, t_2 are equidistant and $t_1 - t_0 = t_2 - t_1 = p$
Substituting $t = t_0$, $t = t_1$, $t = t_2$ in (6.11) respectively, we get :

$$y(t_0) = y(0) = K.A^{B^{t-t_0}} = K.A^{B^0} = K.A \tag{6.12}$$

$$y(t_1) = y(1) = K.A^{B^{t_1-t_0}} \tag{6.13}$$

$$y(t_2) = y(2) = K.A^{B^{t_2-t_0}} \tag{6.14}$$

Applying logarithmic transformations on both sides of equation (6.12), (6.13) and (6.14) we get,

$$ln(y_0) = lnK + lnA \tag{6.15}$$

$$ln(y_1) = lnK + B^P.lnA \tag{6.16}$$

$$ln(y_2) = lnK + B^{2P}.lnA \tag{6.17}$$

From equation (6.15)

$$lnK = lny_0 - lnA$$

$$K = e^{lny_0 - lnA}$$

Subtracting (6.15) from (6.16)

$$lny_1 - lny_0 = lnA(B^p - 1) \tag{6.18}$$

or,

$$\frac{lny_1 - lny_0}{B^p - 1} = lnA$$

or,

$$A = e^{\frac{lny_1 - lny_0}{B^p - 1}}$$

subtracting (6.16) from (6.17)

$$lny_2 - lny_1 = lnA.(B^{2p} - B^p)$$

$$lny_2 - lny_1 = B^p lnA.(B^p - 1) \tag{6.19}$$

Dividing (6.19) by (6.18)

$$\frac{lny_2 - lny_1}{lny_1 - lny_0} = B^P \frac{lnA.(B^p - 1)}{lnA.(B^p - 1)} = B^p$$

or,

$$B^p = (\frac{lny_2 - lny_1}{lny_1 - lny_0})^{\frac{1}{p}}$$

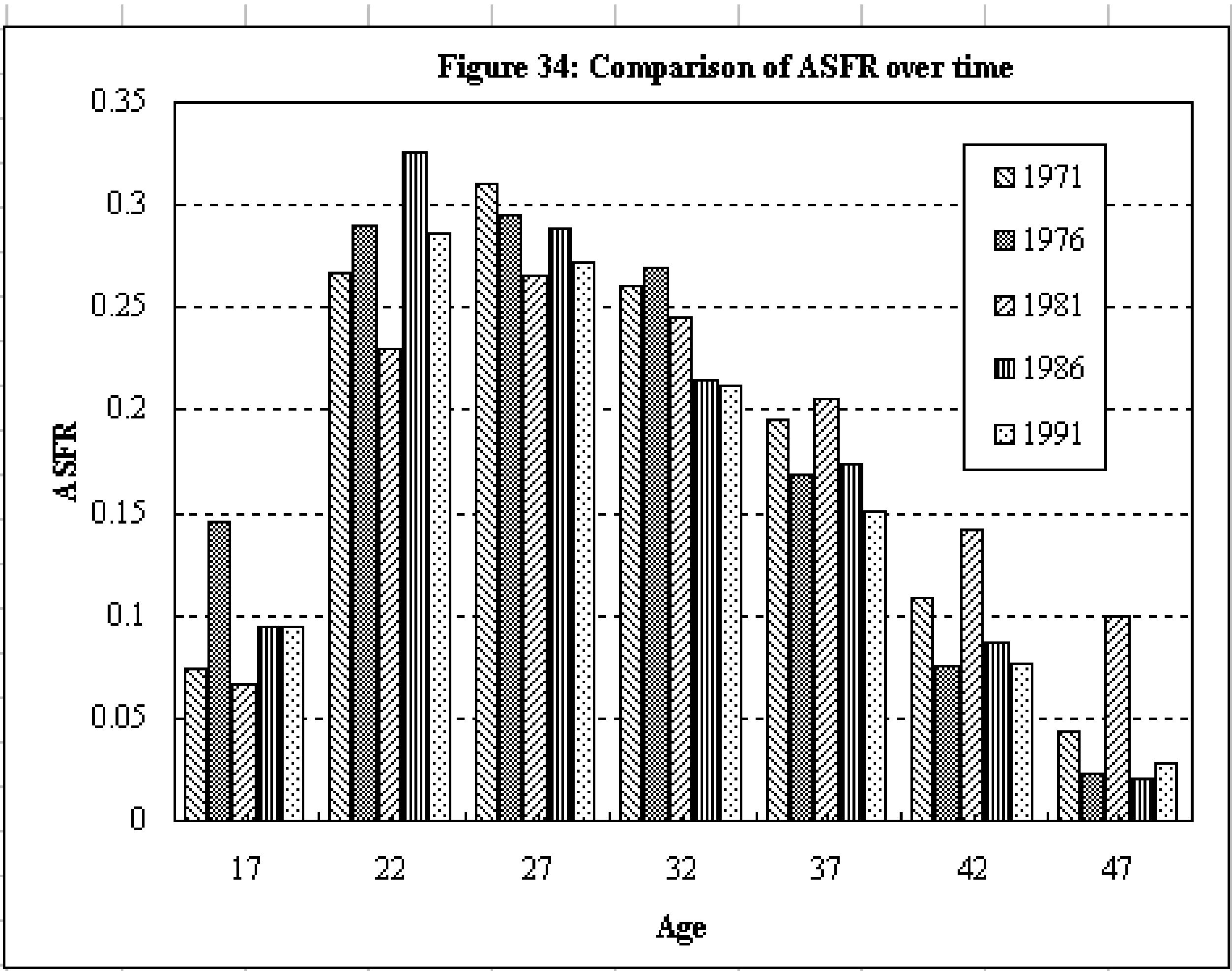
So the starting values of the parameters A, B, K can be found by the following formula.

$$B = (\frac{lny_2 - lny_1}{lny_1 - lny_0})^{\frac{1}{p}}$$

$$A = e^{\frac{lny_1 - lny_0}{B^p - 1}}$$

$$K = e^{lny_0 - lnA}$$

The curve (6.11) is fitted to the cumulative fertility rate.



6.5 Interpretation

While modeling the fertility scene of Nepal, Gamma density showed very good results, in comparison to Hadwiger. Gompertz and Double exponential have also performed well. Low M.S.E and R^2 close to 1 reflect this fact in the Tables 6.2-6.6. Figures 35-47 and Figures 51-55 show the performance of all the four models. Whereas Figures 48-50 show the comparitive performance. The parameters have been projected using linear extrapolation. The projected ASFR values can be seen in Table10.2 and Table10.4. Figure 34 shows the overall behaviour of ASFR over time. It can be seen that the modal age of fertility is gradually increasing.

Figure 35- Comparison between observed and predicted ASFR 1991, Double Exponential model

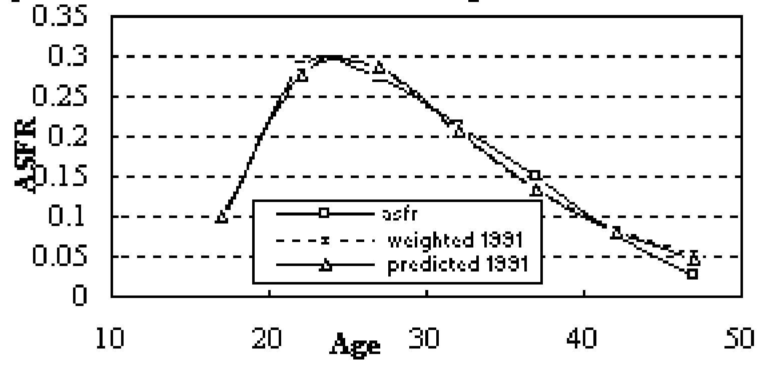


Figure 36- Comparison between observed and predicted ASFR 1986, Double exponential model

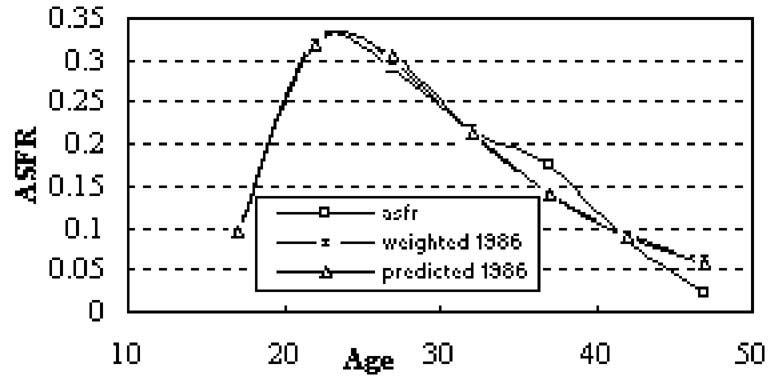


Figure 37 - Comparison between observed and predicted ASFR 1981, Double Exponential model

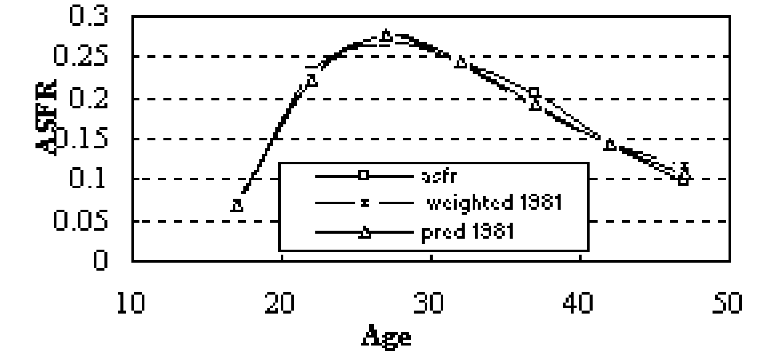


Figure 38 - Comparison between observed and predicted ASFR 1976, Double Exponential model

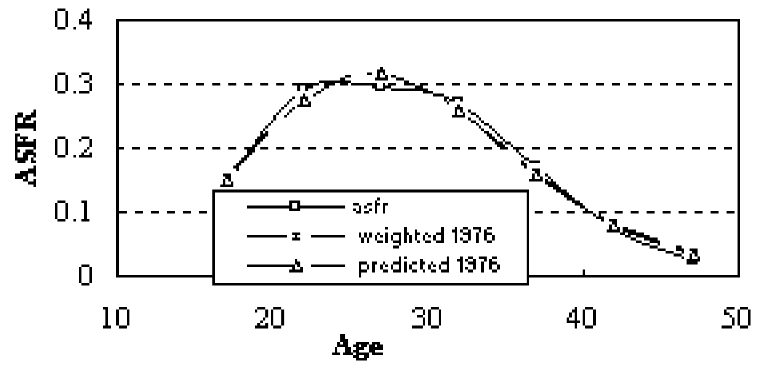


Figure 39 - Comparison between observed and predicted ASFR 1971, Double Exponential model

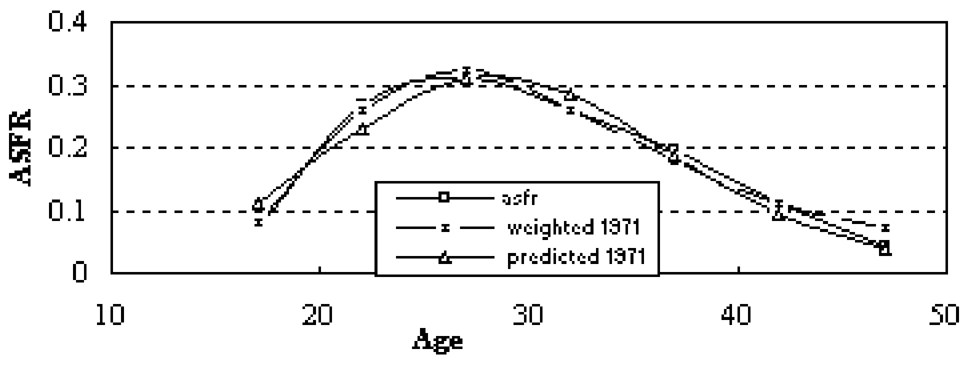


Figure 40 - Comparison between observed and predicted ASFR 1991,Gamma model

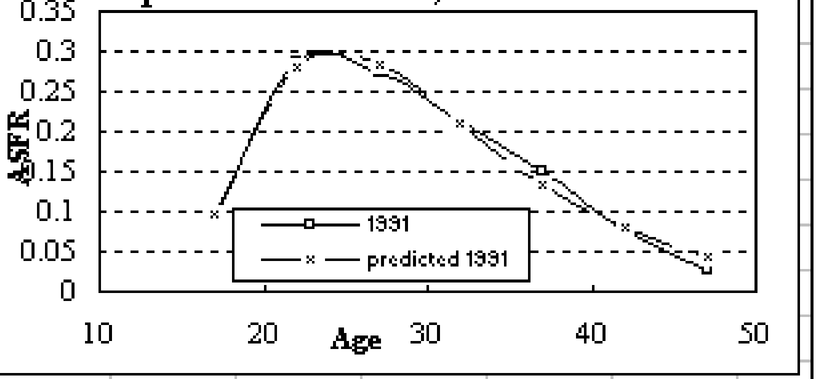


Figure 41 - Comparison between observed and predicted ASFR 1986, Gamma model

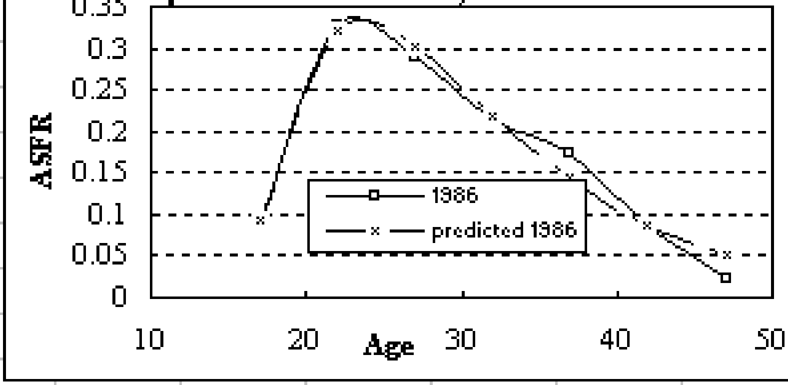


Figure 42 - Comparison between observed and predicted ASFR 1981, Gamma model

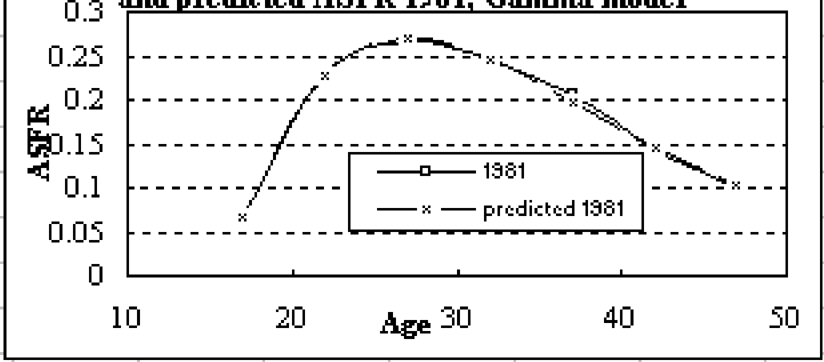


Figure 43 - Comparison between observed and predicted ASFR 1976, Gamma model

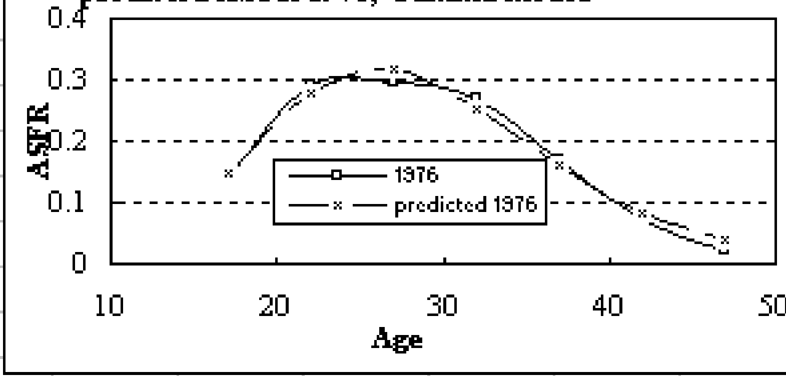


Figure 44 - Comparison between observed and predicted ASFR 1971, Gamma model

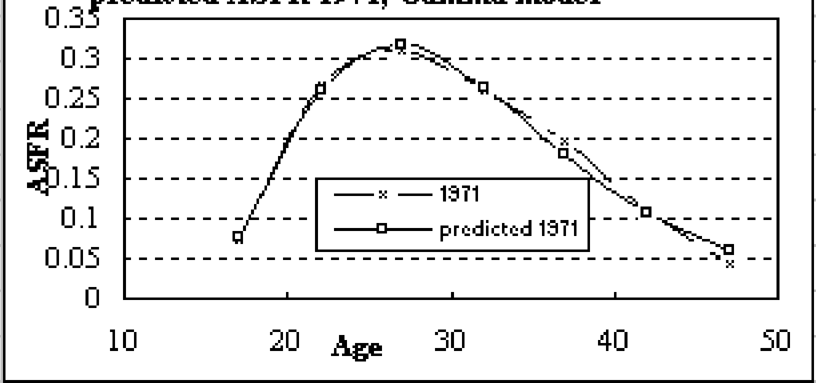


Figure 45 - Comparison between observed and predicted ASFR 1991, Hadwiger model

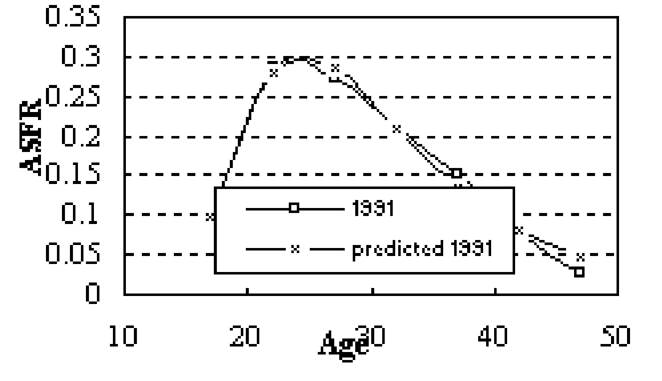


Figure 46 - Comparison between observed and predicted ASFR 1986, Hadwiger model

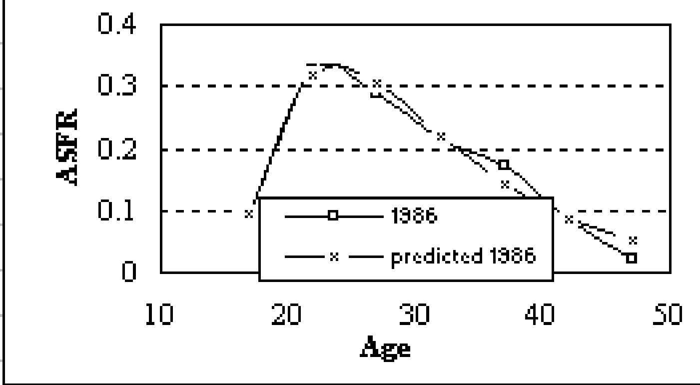


Figure 47 - Comparison between observed and predicted ASFR 1976, Hadwiger model

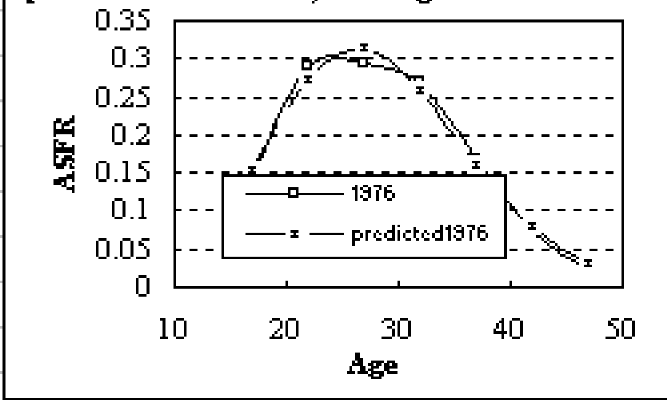


Figure 48 - Comparison between predicted ASFR of four models. 1991

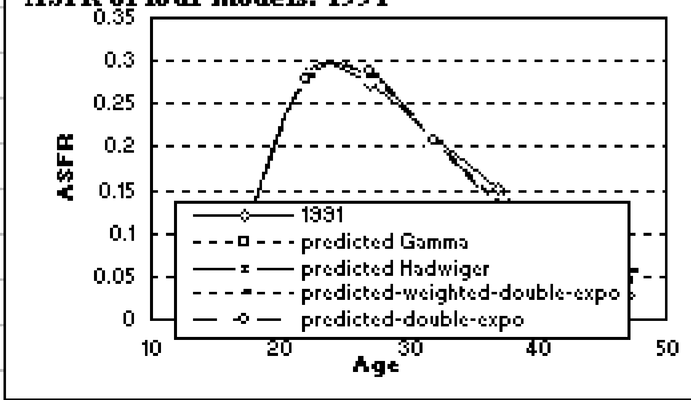


Figure 49 - Comparison between predicted ASFR of four models, 1986

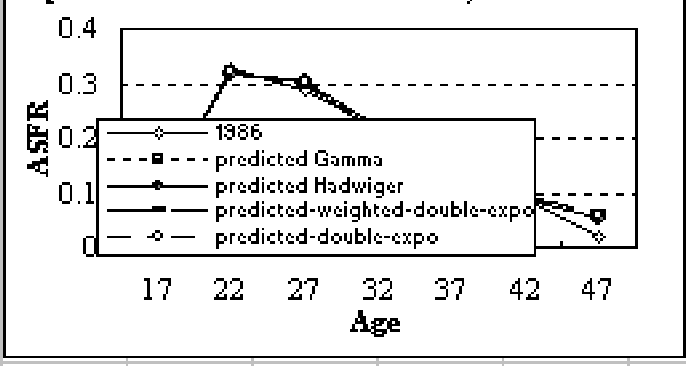


Figure 50 - Comparison between predicted ASFR of four models, 1976

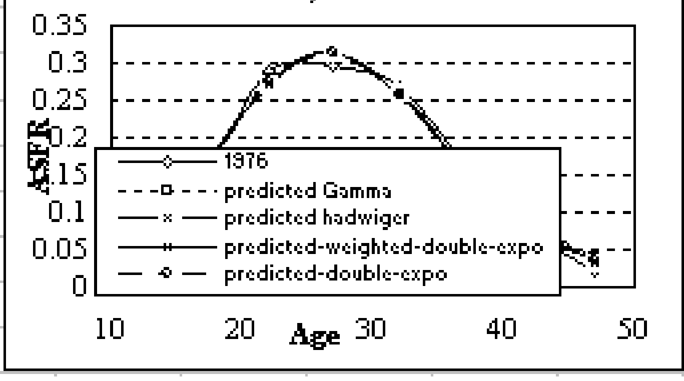


Figure 51 - Comparison between observed an predicted Cumulative ASFR 1991, Gompertz

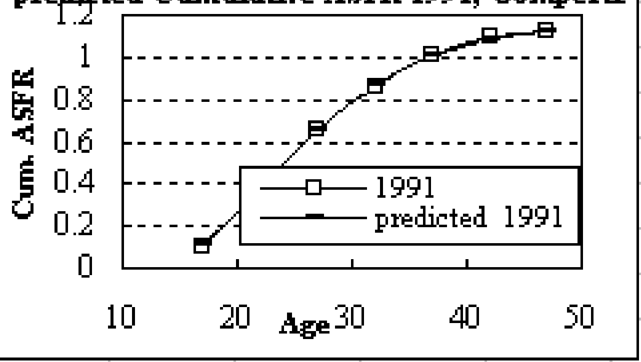


Figure 52 - Comparison between observed and predicted Cumulative ASFR 1986, Gompertz

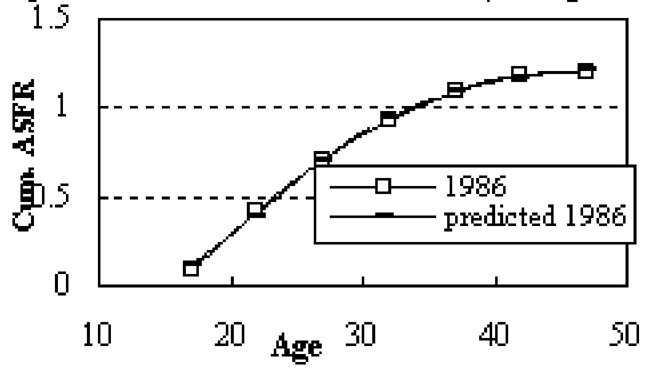


Figure 53 - Comparison between observed and predicted Cumulative ASFR 1981, Gompertz

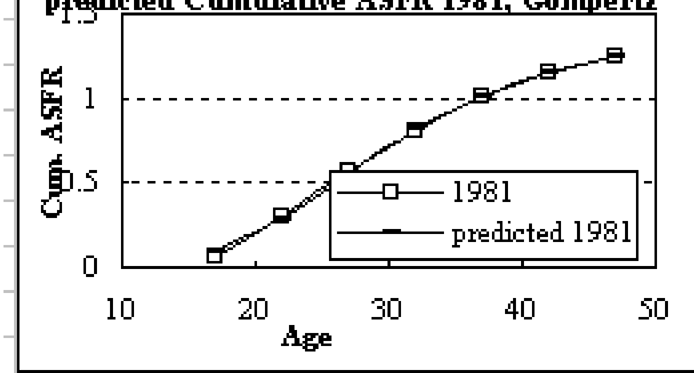


Figure 54 - Comparison between observed an predicted Cumulative ASFR 1976, Gompertz

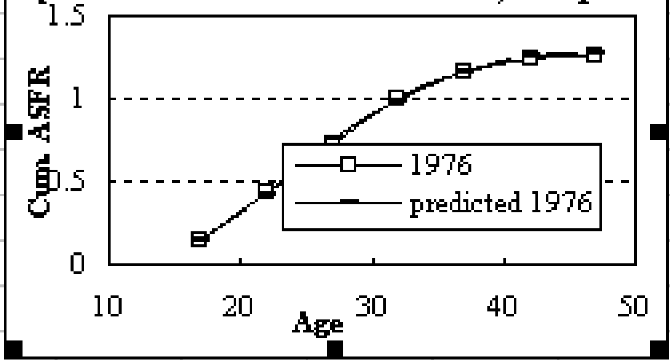


Figure 55 - Comparison between observed and predicted Cumulative ASFR 1971, Gompertz model

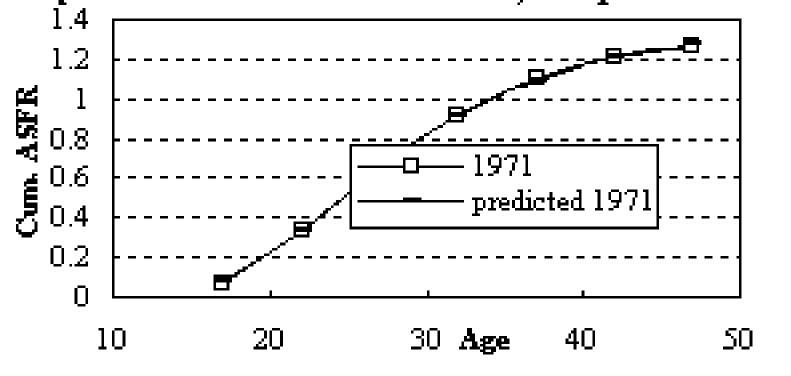


Table 6.2: Double Expo.:Observed and Predicted fertility rates, Nepal 1971-1991

Age	1991		1986		1981	
	Observed	Predicted	Observed	Predicted	Observed	Predicted
15-19	0.095	0.098	0.094	0.096	0.066	0.0698
20-24	0.286	0.278	0.326	0.3196	0.23	0.222
25-29	0.272	0.287	0.289	0.305	0.266	0.277
30-34	0.212	0.207	0.215	0.214	0.245	0.245
35-39	0.151	0.132	0.174	0.139	0.206	0.192
40-44	0.077	0.0800	0.087	0.089	0.142	0.144
45-49	0.028	0.048	0.021	0.056	0.099	0.106

Age	1976		1971	
	Observed	Predicted	Observed	Predicted
15-19	.145	0.150	0.074	0.109
20-24	0.29	0.274	0.267	0.232
25-29	0.295	0.316	0.31	0.311
30-34	0.269	0.257	0.261	0.284
35-39	0.169	0.159	0.196	0.188
40-44	0.075	0.081	0.109	0.095
45-49	0.023	0.035	0.043	0.039

Table 6.3: Comparison between weighted and unweighted Predicted fertility rates, Nepal 1971-1991

Age	1991			1986		
	asfr	weighted	unweighted	asfr	weighted	unweighted
15-19	0.095	0.098	0.098	0.094	0.096	0.096
20-24	0.286	0.282	0.278	0.326	0.323	0.3196
25-29	0.272	0.283	0.287	0.289	0.299	0.305
30-34	0.212	0.204	0.2071	0.215	0.211	0.214
35-39	0.151	0.133	0.132	0.174	0.141	0.139
40-44	0.077	0.084	0.080	0.087	0.094	0.089
45-49	0.028	0.054	0.048	0.021	0.065	0.056

Age	1981			1976		
	asfr	weighted	unweighted	asfr	weighted	unweighted
15-19	0.066	0.071	0.0698	0.145	0.156	0.150
20-24	0.23	0.224	0.222	0.29	0.275	0.274
25-29	0.266	0.278	0.277	0.295	0.315	0.316
30-34	0.245	0.242	0.245	0.269	0.257	0.257
35-39	0.206	0.189	0.192	0.169	0.162	0.1597
40-44	0.142	0.144	0.144	0.075	0.083	0.081
45-49	0.099	0.116	0.106	0.023	0.036	0.035

Age	1971		
	asfr	weighted	unweighted
15-19	0.074	0.082	0.109
20-24	0.267	0.258	0.232
25-29	0.31	0.322	0.311
30-34	0.261	0.2594	0.284
35-39	0.196	0.177	0.188
40-44	0.109	0.1099	0.095
45-49	0.043	0.073	0.039

Table 6.4: Results of Gamma fitting to the A.S.F.R, Nepal

Age	**1991**	**1986**	**1981**	**1971**
R	5.6	6	6.3	6.3
M	28.699374	28.633499	31.453748	29.924603
S^2	55.6090	53.9652	68.3818	56.4665
S	7.4571	7.3461	8.2693	7.5144
mode	22	22	27	27
Initial				
d	14.5	14.5	14.5	14.5
r	5.6	6	6.3	6.3
c	6.699376	6.633499	4.453748	2.924603
b	2.119507	2.130625	3.806625	5.274084
Final				
R	6.511	6.683748	8.15934	7.08345353
B	2.644093	2.081419	2.415338	3.812741
C	5.812354	6.932186	8.442468	4.8009388
D	14.772587	16.005439	15.085	12.9716545
R^2	0.98875	0.97253	0.9878	0.99012
MSE	2.2394E-04	6.9013E-04	1.4554E-04	2.1840E-04

	1991		1986	
Age	Observed	predicted	Observed	predicted
17	0.095	0.095966	0.094	0.094614
22	0.286	0.281158	0.326	0.320886
27	0.272	0.282349	0.289	0.30059
32	0.212	0.20988	0.215	0.219168
37	0.151	0.134997	0.174	0.142983
42	0.077	0.079726	0.087	0.087573
47	0.028	0.044503	0.021	0.051493

	1981		1971	
Age	Observed	predicted	Observed	predicted
17	0.066	0.066646	0.074	0.076374
22	0.23	0.226937	0.267	0.260842
27	0.266	0.271122	0.31	0.317944
32	0.245	0.246235	0.261	0.264461
37	0.206	0.196485	0.196	0.179877
42	0.142	0.145361	0.109	0.108023
47	0.099	0.102324	0.043	0.059603

Table 6.5: Result of Hadwiger fitting to A.S.F.R, Nepal

Age	**1991**	**1986**	**1976**
Initial			
R	5.6	6	6.4
H	21.9561	21.9085	22.5313
T_2	38.3635	38.2803	39.3686
D	38.1515	38.0714	39.1536
Final			
R	5.92762	6.495693	6.7473002
H	1.5265	1.276313	6.520562
T	21.58	19.749947	79.520829
D	8.759098	11.151191	-51.581816
R^2	0.98554	0.96718	0.9849
MSE	2.8802E-04	8.2457E-04	3.5360E-04

	1991		1986		1976	
Age	Observed	predicted	Observed	predicted	Observed	predicted
17	0.095	0.097343	0.094	0.096295	0.074	0.076374
22	0.286	0.279077	0.326	0.31844	0.341	0.260842
27	0.272	0.284909	0.289	0.304307	0.651	0.317944
32	0.212	0.208942	0.215	0.217274	0.912	0.264461
37	0.151	0.133352	0.174	0.140442	1.108	0.179877
42	0.077	0.079555	0.087	0.087259	1.217	0.108023
47	0.028	0.045796	0.021	0.053336	1.26	0.059603

Table 6.6: Result of Gompertz fitting to Cummulative A.S.F.R, Nepal

	1991	**1986**	**1981**	**1976**	**1971**
Initial					
B	0.827458706	0.810509347	0.843590829	0.860293202	0.842008546
A	0.103400063	0.100036289	0.072797666	0.125221248	0.070725666
K	0.91876153	0.939659009	0.906622478	1.15795045	1.046296258
Final					
B	0.868032	0.867425	0.895601	0.872698	0.87423
A	0.094775	0.094931	0.063997	0.1061	0.065043
K	1.162837	1.252469	1.37882	1.332043	1.328708
R^2	0.99918	0.99802	0.99926	0.99901	0.99956
MSE	.0001877	.000529	.0002224	.0002208	.0001386

	1991		1986	
Age	Observed	Predicted	Observed	Predicted
17	0.095	0.110208	0.094	0.118898
22	0.381	0.364103	0.42	0.394078
27	0.653	0.656142	0.709	0.70982
32	0.865	0.877092	0.924	0.947665
37	1.016	1.011959	1.098	1.09217
42	1.093	1.085863	1.185	1.171005
47	1.121	1.124245	1.206	1.211778

	1981		1976		1971	
Age	Observed	Predicted	Observed	Predicted	Observed	Predicted
17	0.066	0.088241	0.145	0.14593	0.074	0.076374
22	0.296	0.282889	0.435	0.279748	0.341	0.260842
27	0.562	0.553533	0.73	0.315413	0.651	0.317944
32	0.807	0.814933	0.999	0.252482	0.912	0.264461
37	1.013	1.018384	1.168	0.159062	1.108	0.179877
42	1.155	1.157929	1.243	0.083957	1.217	0.108023
47	1.254	1.246857	1.266	0.038676	1.26	0.059603

Chapter 7

Parity-progression and Mean age at birth

7.1 Models constructing parity progression functions and maternal age at birth

As mentioned before, fertility has various demographic dimensions. It can be analysed not only through age specific fertility rates but also through parity progression functions and maternal age at birth. This model is unique in the sense that, on the basis of total fertility rate a distribution of maternal fertility could be estimated. It has further lead to the construction of parity progression functions and maternal age at birth[48].

This model gave several useful insights into the fertility scene of Nepal. Once the distribution was estimated, the mean and variance of the distribution helped make a general interpretation about the population under consideration.

Mean Desired Family Size[21]:
More developed of the developing countries have a lower family size as compared to the least developed ones. Increased cost due to children resulting from the socio-economic development, and low economic benefits are responsible for this trend in the developed world. Mean desired family size in the group IV countries (low development) ranges from 3.5 to 8.7. According to the World Fertility Survey the mean desired family size is 3.9 (Nepal) whereas 3.8 (Sri Lanka) 4.2 (Pakistan) and 4.1 (Bangladesh).

Preferred sex of the next child[21]:
The preference of son over daughter is much higher in the south Asian countries (Bangladesh, Nepal, Pakistan).

In each of these countries over 50 percent of the women preferred a son as compared to under 10 percent preferring a daughter. In Pakistan, Nepal and Mauritania, even if all their children are boys, women preferred another son to a daughter. **Proportions wanting no more children[21]:**
WFS has shown that in Asia and Oceania, women were more sensitive to the sex composition of the family when deciding for more children. A family composition with at least one daughter in combination with a number of boys was found to be very popular. In Nepal and Pakistan for example 55-83 percent of those with a daughter and at least two sons were willing to stop as compared with 43-61 percent of those with 3-4 sons and only 5-20 percent of those with no sons. In several other countries like Fiji, Malaysia, Sri Lanka, Syria, Tunisia and Yemen, the contrasts are of same general type but are smaller. In some of these countries, the most satisfactory combination for women with four children is two sons and two daughters, and in others, it is three sons and one daughter. In all these cases women with sons are more likely to have no additional children than women with daughters.

Duration of Breast feeding[21]:
The average period of breast feeding for women in Bangladesh and Nepal is more than 30 months, it is less than 10 months in Fiji and Malaysia. The southern Asian countries of Bangladesh, Nepal, Pakistan, Sri Lanka, and Indonesia have longer average duration of 20 months than other Asian countries. This long duration of breast feeding, similar to that in Africa, is said to have a convex distribution. Thus Asian countries including Nepal, Bangladesh, Pakistan and Sri Lanka have a convex distribution. With economic development, changing pattern of contraceptive uses and availability of breast milk substitute lead to the change from the convex to the concave distribution.Women in low development group (IV) breast feed more than two and a half times more than those in the high development group (I).

Breast feeding is influenced more by the level of development rather than by the family planning effort.

7.1.1 Model Construction

Let $r(x)$ be the instantaneous fertility rate of woman whose age lies in the interval $(x, x+dx]$. Let α and β be the lower and upper limits of the fertility span of woman. Thus $r(x) = 0$, if $x < \alpha$ or $x > \beta$. Total fertility rate is defined as total number children per woman (in her entire fertility span) in a population. Or if the capacity is measured in terms of number of children born, then TFR is the total capacity of a woman to bear a child in her entire fertility span.

thus mathematically

$$CFR = TFR = \int_{\alpha}^{\beta} r(x)dx$$

Cumulative fertility rate, C.F.R upto age x

$$CFR(x) = \int_{\alpha}^{x} r(t)dt$$

Now, an effort is made towards the construction of probability distribution of maternal fertility.

Let X be a random variable representing the number of births to a population of woman (n). Then X has a Poisson distribution [the limiting case of binomial when $n \to \infty$, or when n is very large and $p \to 0$], where p is the instantaneous probability of birth. Then the mean of a random variable during the observation period s is [40],

$$E(X) = ps \tag{7.1}$$

and the variance is

$$varianceX = ps$$

$$P(X=0) = P(ofno.ofbirth) = \frac{e^{-ps}.(ps)^0}{0!} = e^{-ps} \tag{7.2}$$

Let us compare it with the situation where instantaneous probability of birth at time a (time dependent) is $r(a)$. When the observation period y is such that $\alpha < y < x$.

$$E(X) = \int_{\alpha}^{x} r(t)dt$$

from (7.1)

From (7.2) probability of no birth in the observation period y, $\alpha < y < x$ is

$$P(X=0) = e^{-\int_{\alpha}^{x} r(t)dt} \tag{7.3}$$

Here the observation time is continuous.

Let $f_1(x)dx$ be the instantaneous probability of experiencing first birth in the age interval $(x, x+dx]$.

$f_1(x)$ =P(of first birth in the time interval $(x, x+dx]$) =P(no birth before age x).P(instantaneous birth in $(x, x+dx]$) $= e^{-\int_{\alpha}^{x} r(t)dt}.r(a)$
from (7.3), $\alpha \leq x < \beta$

$F_1 = \int_\alpha^\beta f_1(x)dx$ =P[of having at least 1 child in (α,β)]=1-P[of having no children in (α,β)]

$$= 1 - e^{-\int_\alpha^\beta r(x)dx}$$

$$= 1 - e^{-CFR}$$

The probability density function of age at first birth, for all those women who ever bear a child is calculated as follows.

P(a woman is aged x| a woman has first child)=
P(a woman is aged x And a woman has first child)/P(a woman has first child) $h_1(x) = \frac{f_1(x)}{F_1}$
$h_1(x) = \frac{1}{F_1}.r(x).e^{-\int_a^x r(t)dt}$ where $\alpha \le x < \beta$

$$A_1 = \int_\alpha^\beta x.h_1(x)dx$$

Where A_1 is the mean age at first birth.

$$A_1 = \frac{1}{F_1}.\int_\alpha^\beta x.f_1(x)dx$$

$$= \frac{1}{F_1}.\int_\alpha^\beta x.r(x).e^{-\int_\alpha^x r(t)dt}dx$$

$$A_1 = \frac{1}{F_1}.\int_\alpha^\beta x.f_1(x)dx$$

$$\frac{d}{dx}e^{-\int_\alpha^x r(t)dt} = -r(x).e^{-\int_\alpha^x r(t)dt}$$

The result in the expression above can be explained as following; let

$$\int_\alpha^x r(t)dt = R(x) - R(\alpha)$$

where

$$\int r(a)da = R(a)$$

then

$$\frac{d}{dx}e^{-\int_\alpha^x r(t)}dt = \frac{d}{dt}e^{-(R(x)-R(\alpha))}$$

$$= -r(x).e^{-\int_\alpha^x r(t)dt}$$

so,

$$A_1 = \frac{1}{F_1}.\int_\alpha^\beta x.\frac{d}{dx}[e^{-\int_\alpha^x r(t)dt}]dx$$

Integrating the right hand side by parts,

$$A_1 = \frac{1}{F_1}.[x.(e^{-\int_\alpha^x r(t)dt})|_\alpha^\beta - \int_\alpha^\beta -e^{-\int_\alpha^x r(t)dt}]$$

$$= \frac{1}{F_1}[-\beta.e^{-\int_\alpha^\beta r(t)dt} + \alpha.e^{-\int_\alpha^\alpha r(t)dt} + \int_\alpha^\beta e^{-\int_\alpha^x r(t)dt}]$$

$$= \frac{1}{F_1}[-\beta.e^{-\int_\alpha^\beta r(t)dt} + \alpha.e^0 + \int_\alpha^\beta e^{-\int_\alpha^x r(t)dt}]$$

$$\int_\alpha^\beta r(a)da = TFR$$

$$\int_\alpha^x r(a)da = TFR(x)$$

$$= \frac{1}{F_1}[-\beta.e^{-TFR} + \alpha + \int_\alpha^\beta e^{-TFR(x)}dx]$$

$$F_1 = 1 - e^{-TFR}$$

from the derivation above.

$$F_1 + e^{-TFR} = 1$$

so,

$$A_1 = \frac{1}{F_1}[-\beta.e^{-TFR} + \alpha[F_1 + e^{-TFR}] + \int_\alpha^\beta e^{-TFR(x)}dx]$$

$$= \alpha + \frac{1}{F_1}.[((\alpha - \beta).e^{-CFR} + \int_\alpha^\beta .e^{-CFR(x)}dx)]$$

This result is same as the result obtained by Pandey [48]. Now the instantaneous probability of second birth in the age interval $(y, y + dy]$.
P(second birth in the interval $(y, y+dy]$)=P(no birth before x).P(single birth in $(x, x + dx]$).P(no birth in $(x + dx, y]$).P(single birth in $(y, y + dy]$)

$$= e^{-\int_\alpha^x r(t)dt}.r(x).e^{-\int_x^y r(t)dt}.r(y)$$

$= f_1(x).r(y).e^{-\int_x^y r(t)dt}$ where $\alpha \leq x \leq y$
$f_2(y)$is the probability of second birth in the interval $(y, y + dy]$ such that $\alpha \leq x \leq y$

$$f_2(y) = \int_\alpha^y e^{-\int_x^y r(t)dt}.r(y).f_1(x).dx$$

$$F_2(y) = \int_\alpha^\beta f_2(y)dy$$

=P(of giving birth to at least two children)
=1−P(no birth)−P(one birth)
$= 1 - e^{-CFR} - (CFR).e^{-CFR}$ from (7.2) and (7.3). where

$$CFR = \int_\alpha^\beta r(x)dx$$

Similarly $h_2(x)$ is the probability density of age at second birth.
$h_2(x)$ =P(a woman is age x|a woman has two children)
=P(a woman is aged x And a woman has two children)
$= \frac{f_2(x)}{F_2}$. Similarly A_2 is the mean age at second birth. It is calculated as following;

$$A_2 = \int_\alpha^\beta x.h_2(x) = \int_\alpha^\beta x.\frac{f_2(x)}{F_2(x)}dx$$

$$= \frac{1}{F_2}.\int_\alpha^\beta x.r(x).e^{-\int_\alpha^x r(t)dt}.(\int_\alpha^x r(a)da)dx$$

$$\frac{d}{dx}[(\int_\alpha^x r(a)da).(e^{-\int_\alpha^x r(t)dt}) + e^{-\int_\alpha^x r(t)dt}]$$

$$= r(x).e^{-\int_\alpha^x r(t)dt} - (\int_\alpha^x r(a)da).(e^{-\int_\alpha^x r(t)dt}).r(x) - r(x).e^{-\int_\alpha^x r(t)dt}$$

$$= (-\int_\alpha^x r(a)da).(e^{-\int_\alpha^x r(t)dt}).r(x)$$

so,

$$A_2 = \frac{1}{F_2}\int_\alpha^\beta x.\frac{d}{dx}[-(\int_\alpha^x r(a)da).(e^{-\int_\alpha^x r(t)dt}) - e^{-\int_\alpha^x r(t)dt}]$$

$$= \frac{1}{F_2}[x[-(\int_\alpha^x m(a)da).e^{-\int_\alpha^x r(t)dt} - e^{-\int_\alpha^x r(t)dt}]|_\alpha^\beta + \int_\alpha^\beta TFR(x).e^{-TFR(x)}dx]$$

$$F_2 = 1 - e^{-TFR}.\sum_{j=0}^{1}\frac{(TFR)^j}{j!}$$

or,

$$1 = F_2 + e^{-TFR}.\sum_{j=0}^{1}\frac{(TFR)^j}{j!}$$

Substituting instead of 1, the RHS of the above expression

$$A_2 = \frac{1}{F_2}[\alpha(0+1) - \beta[TFR.e^{-TFR} + e^{-TFR}]] + \frac{1}{F_2}[\int_\alpha^\beta [TFR(x)e^{-TFR(x)} + e^{-TFR(x)}]]$$

$$A_2 = \alpha + \frac{1}{F_2}[(\alpha-\beta)[\sum_{j=0}^{1} e^{TFR}.\frac{(TFR)^j}{j!}] + \int_\alpha^\beta e^{-TFR(x)}.\sum_{j=0}^{1}\frac{(TFR(x))^j}{j!}]$$

Similarly $h_i(x) = \frac{f_i(x)}{F_i}$, where $i = 1, 2, ...$

$$F_i = 1 - e^{-CFR}.\sum_{j=0}^{i-1}\frac{(CFR)^j}{j!} \tag{7.4}$$

$$f_i(x) = \int_\alpha^x r(x).e^{-\int_x^z r(t)dt}.f_{i-1}(z)dz$$

similarly A_i can be calculated as A_1,A_2 have been calculated.

$$A_i = \int_\alpha^\beta x.h_i(x)$$

$$= \alpha + \frac{1}{F_i}[(\alpha-\beta)[\sum_{j=0}^{i-1} e^{TFR}.\frac{(TFR)^j}{j!}] + \int_\alpha^\beta e^{-TFR(x)}.\sum_{j=0}^{i-1}\frac{(TFR(x))^j}{j!}]$$

Where a woman gave birth to the ith child in the age group $(x, x + dx]$ **Parity Progression tables:** They are analogous to life tables. **The first column** is occupied by the **i** parity, number of children ($i = 1, 2, 3, ..$) born to a woman. **The second column** corresponds to l_x in the life table, it shows number F_i of women passing through parity i among F_0 women all initially of parity 0. **The third column** gives $F_i - F_{i+1}$, that is the number of women having exactly i children. This column corresponds to the d_x column of the life table. **The fourth column** gives the $a_i = \frac{F_i}{F_{i-1}}$, which represents the probability of moving from parity $i-1$ to parity i [51]. The irregularity and sensitivity among and within the population are reflected very effectively by Parity progression ratio.

From the Age specific fertility rates data of Nepal, the parity progression table was constructed. The derivative of probability distribution, its mean and variance led to the construction of the former. The construction of parity progression tables and mean age at first and consequent births has highlighted several important features of Nepalese fertility and Indian fertility.

7.1.2 Model Application

The age specific fertility data[11] are for discrete ages. But the formula above are derived for continuous age (time) .

$$TFR = \int_\alpha^\beta r(x)dx$$

$$CFR(x) = \int_{\alpha}^{x} r(x)dx$$
$$= TFR(x)$$

if m_j is the age specific fertility data in the age group $(a_j, a_{j+1}]$ then

$$CFR = TFR = \sum_{j=1}^{n}(a_{j+1} - a_j)m_j$$

That is m_j is assumed to be constant in the interval $(a_j, a_{j+1}]$

$$CFR(x) = \int_{\alpha}^{x} r(x)dx$$

$$= \sum_{k=1}^{j}(a_{j+1} - a_j)m_k - (a_{j+1} - x_j{}^*)m_j$$

Where $x_j{}^*$ is the mid point of $(a_j, a_{j+1}]$
TFR for example, for year 1991, for Nepal 5.6 thus

$$F_i = 1 - e^{-TFR}\sum_{j=0}^{i-1}\frac{(TFR)^j}{j!} \tag{7.5}$$

Substituting the values of TFR and i in (7.5) we get:

$$F_0 = 1$$
$$F_1 = 1 - e^{-5.6} = .996302$$
$$F_2 = 1 - e^{-5.6}[1 + 5.6] = .9755941$$
$$F_3 = 1 - e^{-5.6}[1 + 5.6 + \frac{(5.6)^2}{2!}] = .9176116$$
$$\vdots$$
$$\vdots$$
$$F_8 = 1 - e^{-5.6}[1 + 5.6 + \frac{(5.6)^2}{2!}] + \frac{(5.6)^3}{3!} + \cdots + \frac{(5.6)^7}{7!} = .3297422$$

$$p_i = \frac{F_{i+1}}{F_i} \tag{7.6}$$

$$B_i = F_i - F_{i+1} \tag{7.7}$$

Then by substituting the calculated values of F_i, $i = 0, 1, 2, \cdots, 8$ in (7.6) and (7.7) the values of p_i and B_i were obtained. The results can be seen in the Table 7.1.

7.1.3 Calculation of A_1, A_2, A_3,$\cdots$, A_i

$$A_1 = \alpha + \frac{1}{F_1}.[((\alpha-\beta).e^{-CFR} + \int_\alpha^\beta .e^{-CFR(x)}dx)] \tag{7.8}$$

$$TFR = CFR = \sum_{j=1}^{n}(a_{j+1}-a_j)m_j = \int_\alpha^\beta r(x)dx$$

$$TFR(x) = CFR(x) = \int_\alpha^x r(x)dx$$

$$= \sum_{i=1}^{j}(a_{i+1}-a_i)m_i - (a_{j+1}-x_j{}^*)m_j$$

$$e^{-CFR(x)} = e^{-\sum_{i=1}^{j}(a_{i+1}-a_i)m_k-(a_{j+1}-x_j{}^*)m_j}$$

$$\int_\alpha^\beta e^{-CFR(x)}dx = \sum_{j=1}^{k}(a_{j+1}-a_j)f(x_j)$$

$$f(x_j) = e^{-\sum_{i=1}^{j}(a_{i+1}-a_i)m_i+(a_{j+1}-x_j{}^*)m_j}$$

The value of $\int_\alpha^\beta e^{-CFR(x)}dx$, has been calculated. It is placed in equation (7.8) and A_1 is calculated.

$$\int_\alpha^\beta e^{-CFR(x)}dx = \sum_{j=1}^{k}(a_{j+1}-a_j)f(x_j)$$

$$f(x_j) = e^{-\sum_{i=1}^{j}(a_{i+1}-a_i)m_i+(a_{j+1}-x_j{}^*)m_j}$$

The right hand side is calculated with the help of EXCEL-work sheets. The basic idea of how this worksheet calculation has been done, is mentioned briefly below.

For the calculation of A_1 the summand e^{-TFR} can be calculated without much problem. The calculation of the summand

$$\int_\alpha^\beta e^{-CFR(x)}dx = \sum_{j=1}^{k}(a_{j+1}-a_j)f(x_j)$$

where,

$$f(x_j) = e^{-\sum_{i=1}^{j}(a_{i+1}-a_i)m_i+(a_{j+1}-x_j{}^*)m_j}$$

, is a bit complicated. In order to do the worksheet calculations without much complication, some simplification in this summand have been made. $a_{j+1}-a_j = 5$, $\forall\ j = 1, 2, \cdots, 7$ taking Nepalease data into consideration.

$$= (a_2-a_1)EXP[-(a_2-a_1)m_1 + (a_2-x_1)m_1]|_{j=1}+$$

$$+(a_3-a_2)EXP[-\sum_{i=1}^{2}(a_{i+1}-a_i)m_i+(a_3-x_2)m_2]|_{j=2}+$$

$$+(a_4-a_3)EXP[-\sum_{i=1}^{3}(a_{i+1}-a_i)m_i+(a_4-x_3)m_3]|_{j=3}+$$

$$+(a_5-a_4)EXP[-\sum_{i=1}^{4}(a_{i+1}-a_i)m_i+(a_5-x_4)m_4]|_{j=4}+$$

$$\vdots$$

$$+(a_8-a_7)EXP[-\sum_{i=1}^{7}(a_{i+1}-a_i)m_i+(a_8-x_7)m_7]|_{j=7}$$

Th preceding expression simplified the calculation of summand 2 of A_1

Similar steps were taken, which helped facilitate the simplification of calculation of A_2, A_3,... let

$$-(a_2-a_1)m_1+(a_2-x_1)m_1=T_1$$

$$-\sum_{i=1}^{2}(a_{i+1}-a_i)m_i+(a_3-x_2)m_2=T_2$$

$$\vdots$$

$$-\sum_{i=1}^{k}(a_{i+1}-a_i)m_i+(a_{k+1}-x_k)m_k=T_k$$

$$A_i=\alpha+\frac{1}{F_i}[(\alpha-\beta)[\sum_{j=0}^{i-1}e^{TFR}.\frac{(TFR)^j}{j!}]+\int_\alpha^\beta e^{-TFR(x)}.\sum_{j=0}^{i-1}\frac{(TFR(x))^j}{j!}]$$

The summand

$$\int_\alpha^\beta e^{-TFR(x)}.\sum_{j=0}^{i-1}\frac{(TFR(x))^j}{j!}$$

is simplified as following: when say $i=2$, the summand is

$$\int_\alpha^\beta e^{-TFR(x)}.\sum_{j=0}^{1}\frac{(TFR(x))^j}{j!}$$

$$=\sum_{j=1}^{k}(a_{j+1}-a_j)EXP[-\sum_{i=1}^{j}-(a_{i+1}-a_i)m_i+(a_{j+1}-x_j)m_j]$$

$[1+\sum_{i=1}^{j}(a_{i+1}-a_i)m_i-(a_{j+1}-x_j)m_j]$

The summand of A_r is simplified to

$$(a_2-a_1)e^{T_1}[1+T_1+\frac{(T_1)^2}{2!}+\frac{(T_1)^3}{3!}+\cdots+\frac{(T_1)^r}{r!}]|_{j=1}+$$

$$(a_3-a_2)e^{T_2}[1+T_2+\frac{(T_2)^2}{2!}+\frac{(T_2)^3}{3!}+\cdots+\frac{(T_2)^r}{r!}]|_{j=2}+$$

$$\vdots$$

$$(a_{k+1}-a_k)e^{T_k}[1+T_k+\frac{(T_k)^2}{2!}+\frac{(T_k)^3}{3!}+\cdots+\frac{(T_k)^r}{r!}]|_{j=k}+$$

7.1.4 Interpretation

As seen from the figures, Parity progression ratios for developing countries are regular and decline monotonically from a maximum at parity zero to minimum at highest parity.

In a low-fertility industrialized countries (Portugal and Netherlands) there is a steep decline in parity progression ratios until parity two, after which the curve levels off or even increases. The reason behind this behaviour is that a small group of women with extremely high fertility dominates beyond a certain parity. In Netherlands the parity progression ratio shows a steep decline from parity two to three, but a slight increase to parity four [37].

7.1.5 Results

Here the parity progression tables for Nepal (also seee Figure 56)as well as India (also see Figures 61-62) were constructed. And then the results have been compared (also see Figures 57-58). For Nepal the TFR (Total Fertility rate) and ASFR (Age Specific Fertility rate) data were obtained from Population Monograph of Nepal [11]. For India, two type of data were used, adjusted and unadjusted. Adjusted TFR and ASFR data were obtained from Regression estimates of fertility for India 1971 and 1981 [52]. The unadjusted data were obtained from Occasional Papers, Fertility of India [25]. After citing the results, a comparison was made between India and Nepal. In addition, a parity progression table has been constructed for Bihar, a north indian state, which has a common border with Nepal. It was very interesting to establish a comparison between the results of Bihar and Nepal and U.P (also see Figures 59-60) .

The tables on mean age at birth reflected, that the mean age at first birth is increasing for Nepal. It increased from 19 years in 1971 to 21 years

Table 7.1: Method of construction of parity progression table

TFR(1991)	5.6
F_0	1
F_1	0.9963
$1 + TFR$	6.6
F_2	0.9756
$1 + TFR + (TFR^2/2)$	22.28
F_3	0.9176
$1 + TFR + (TFR^2/2) + (TFR^3/6)$	51.5493
F_4	0.8094
$1 + TFR + (TFR^2/2) + (TFR^3/6) + (TFR^4/24)$	92.5264
F_5	0.6578
$1 + TFR + (TFR^2/2) + (TFR^3/6) + (TFR^4/24)$ $+(TFR^5/120)$	138.4207
F_6	0.4881
$1 + TFR + (TFR^2/2) + (TFR^3/6) + (TFR^4/24)$ $+(TFR^5/120) + (TFR^6/720)$	181.2554
F_7	0.3297
$1 + TFR + (TFR^2/2) + (TFR^3/6) + (TFR^4/24)$ $+(TFR^5/120) + (TFR^6/720) + (TFR^7/5040)$	215.5232
F_8	0.2030

in 1991. The mean age at first birth for India for the year 1981 is 21.7 years as compared to 20 years for Nepal. These Tables 7.2-7.6 reflect the fact that various family planning programms are showing results. When the decline in fertility is considered, the urban areas in India are far ahead of the rural areas.

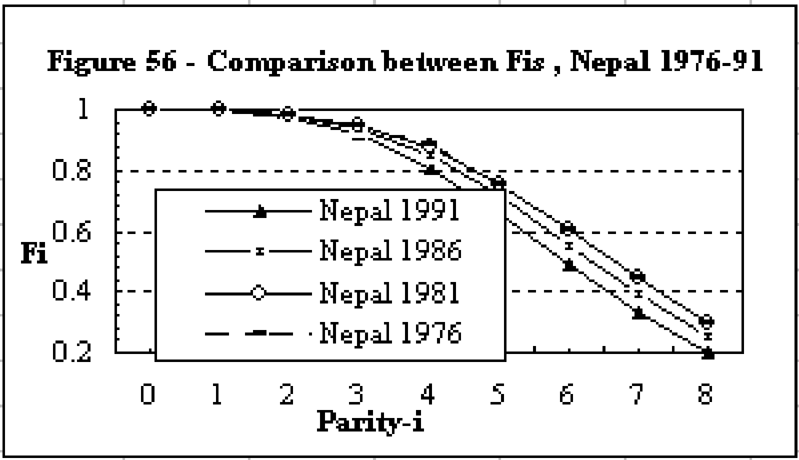


Figure 56 - Comparison between Fis , Nepal 1976-91

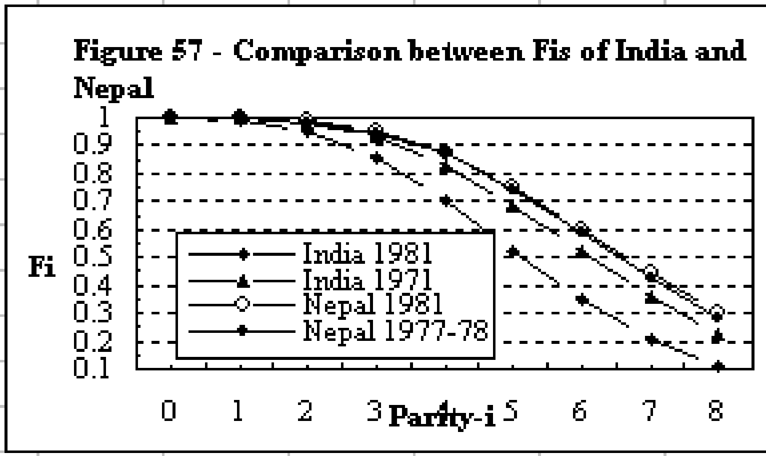


Figure 57 - Comparison between Fis of India and Nepal

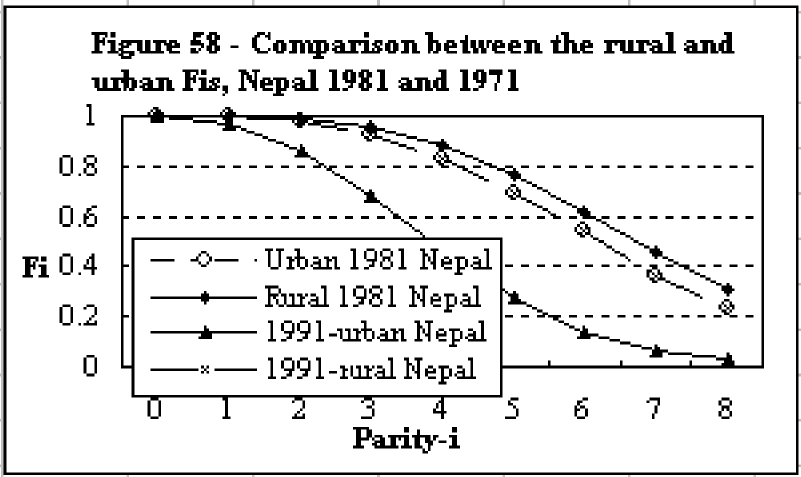


Figure 58 - Comparison between the rural and urban Fis, Nepal 1981 and 1971

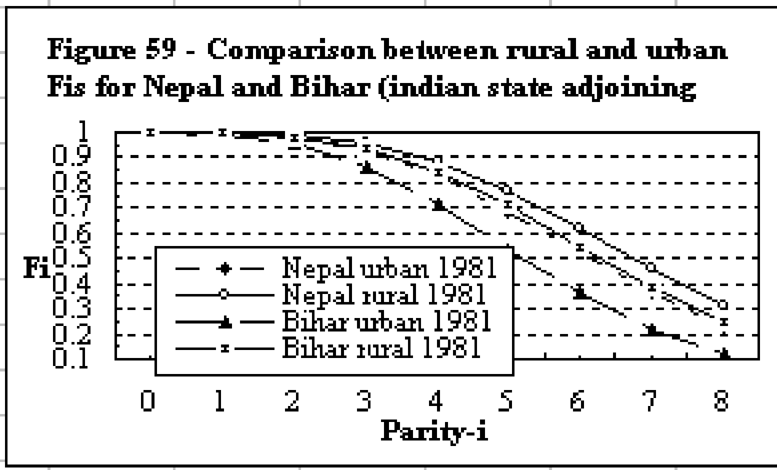


Figure 59 - Comparison between rural and urban Fis for Nepal and Bihar (indian state adjoining

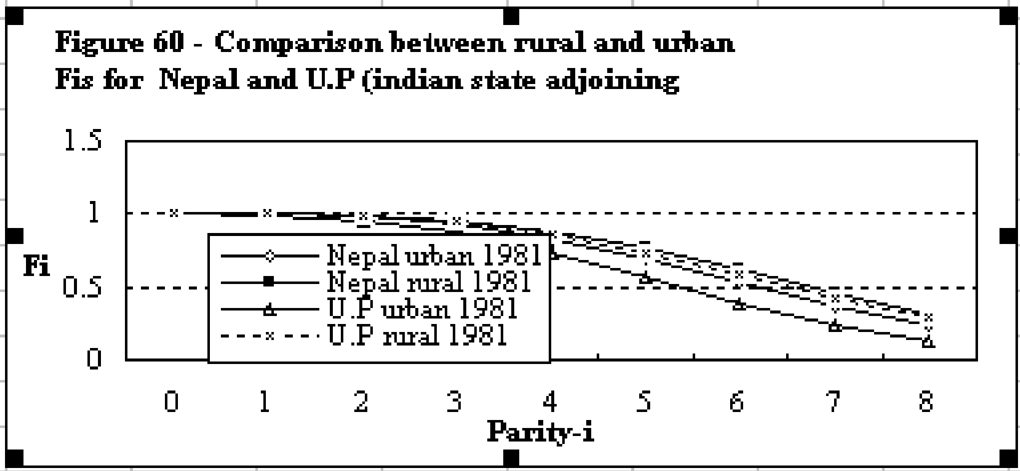


Figure 60 - Comparison between rural and urban Fis for Nepal and U.P (indian state adjoining

Figure 61 - Comparison between the Fis of India whole and Bihar (rural and urban), 1971 and 1981

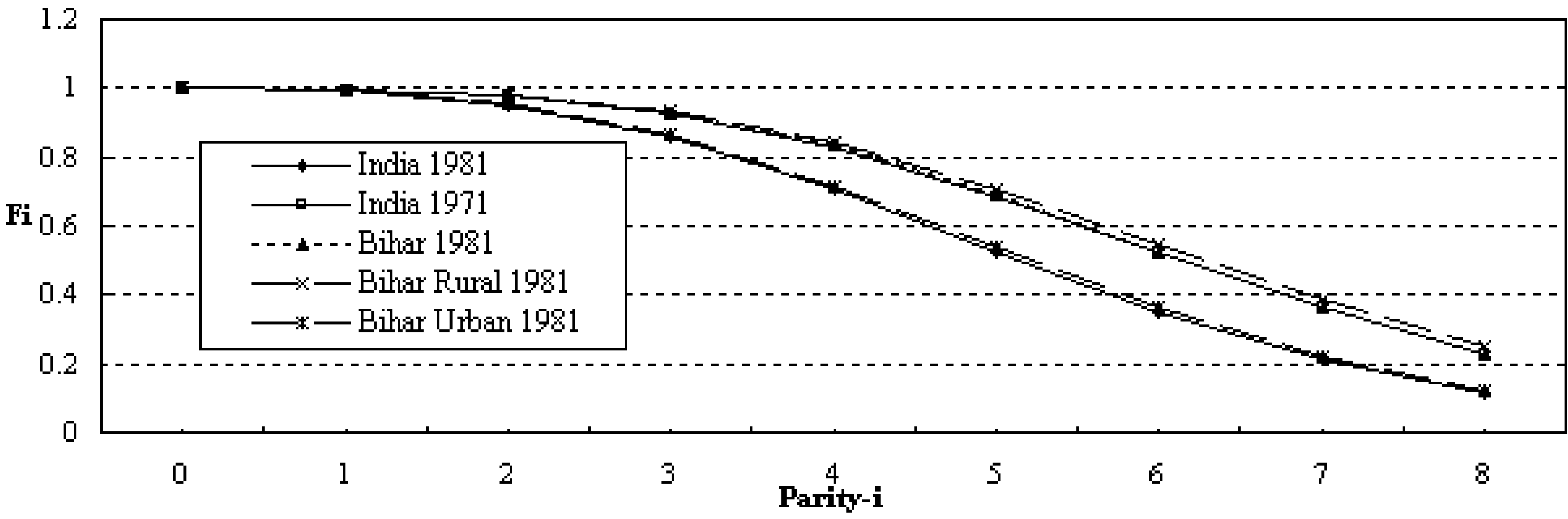


Figure 62 - Comparison between the Fis of Bihar whole, Bihar rural and Bihar Urban.

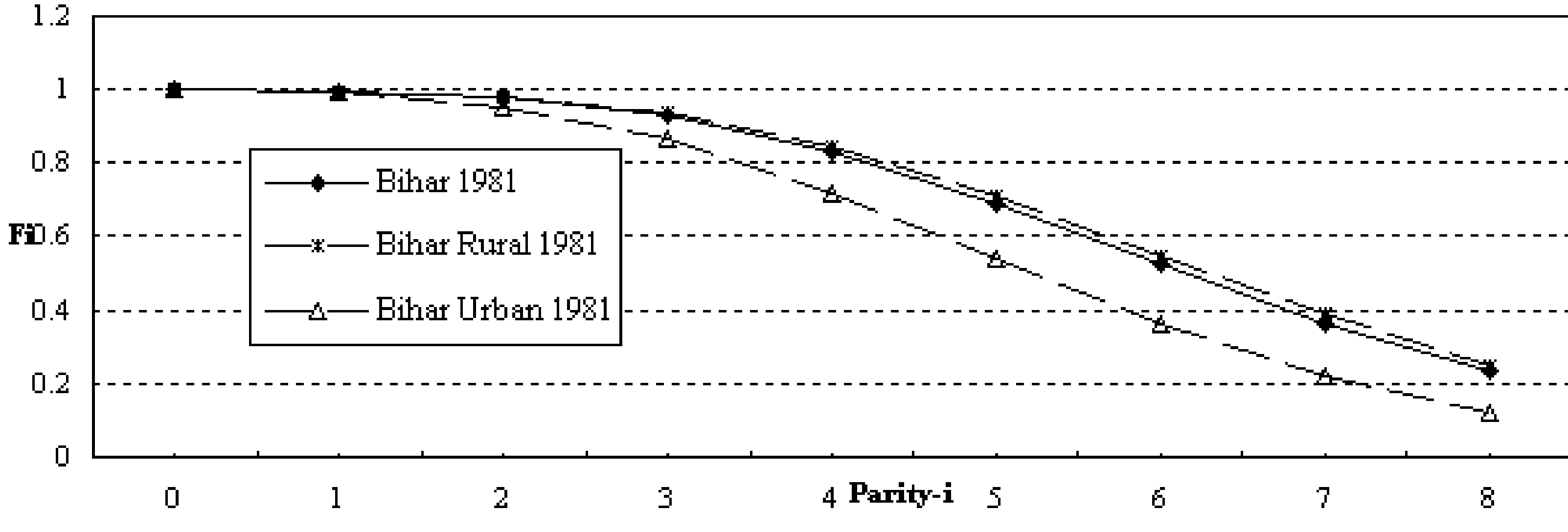


Table 7.2: The Parity Progression Table Nepal

1991			
Parity i	F_i	p_i	B_i
0	1	0.996302136	0.003697864
1	0.996302136	0.979215103	0.020708037
2	0.975594099	0.940566981	0.057982503
3	0.917611596	0.882048128	0.108234006
4	0.809377591	0.812785022	0.151527608
5	0.657849983	0.742021851	0.169710921
6	0.488139062	0.675508739	0.15839686
7	0.329742202	0.615707401	0.126717488
8	0.203024714		0.203024714
average Family size	5.377641383		
1986			
Parity i	F_i	p_i	B_i
0	1	0.997521248	0.002478752
1	0.997521248	0.98509053	0.014872513
2	0.982648735	0.95459462	0.044617539
3	0.938031196	0.904869818	0.089235078
4	0.848796117	0.842302981	0.133852618
5	0.7149435	0.775334497	0.160623141
6	0.554320359	0.710234094	0.160623141
7	0.393697218	0.650297305	0.137676978
8	0.25602024		0.25602024
average Family size	5.685978611		
1981			
Parity i	F_i	p_i	B_i
0	1	0.998163695	0.001836305
1	0.998163695	0.988409997	0.01156872
2	0.986594975	0.963063396	0.036441468
3	0.950153507	0.91945819	0.076527083
4	0.873626423	0.862034672	0.120530156
5	0.753096267	0.798341854	0.151867997
6	0.60122827	0.734773954	0.159461397
7	0.441766873	0.675133501	0.143515257
8	0.298251616		0.298251616
average Family size	5.902881626		

1977/78			
Parity i	F_i	p_i	B_i
0	1	0.997970569	0.002029431
1	0.997970569	0.987391943	0.01258247
2	0.985388099	0.960415945	0.039005657
3	0.946382443	0.914821232	0.080611691
4	0.865770752	0.855679901	0.124948121
5	0.740822631	0.790859967	0.15493567
6	0.585886961	0.72673877	0.160100192
7	0.425786769	0.666962345	0.141803027
8	0.283983742		0.283983742
average Family size	5.831991967		

Table 7.3: The Parity Progression India adjusted

1981			
Parity i	F_i	p_i	B_i
0	1	0.991770253	0.008229747
1	0.991770253	0.960169419	0.039502786
2	0.952267467	0.900441116	0.094806686
3	0.857460781	0.823093136	0.151690698
4	0.705770084	0.742084793	0.182028837
5	0.523741246	0.666347295	0.174747684
6	0.348993563	0.599424855	0.139798147
7	0.209195416	0.541760578	0.095861586
8	0.113333829		0.113333829
average Family size	4.702532639		

1971			
Parity i	F_i	p_i	B_i
0	1	0.996972445	0.003027555
1	0.996972445	0.982386858	0.017559818
2	0.979412628	0.948006112	0.050923471
3	0.928489157	0.893965328	0.098452044
4	0.830037113	0.828013156	0.142755463
5	0.68728165	0.759056077	0.165596337
6	0.521685313	0.693155135	0.160076459
7	0.361608853	0.633209255	0.132634781
8	0.228974073		0.228974073
average Family size	5.534461232		

Table 7.4: The Parity Progression Bihar Adjusted

1981			
Parity i	F_i	p_i	B_i
0	1	0.997032395	0.002967605
1	0.997032395	0.982677131	0.017271462
2	0.979760933	0.948701818	0.050259954
3	0.929500979	0.895100367	0.097504311
4	0.831996667	0.829483965	0.141868773
5	0.690127895	0.7607179	0.165135252
6	0.524992643	0.694888688	0.160181194
7	0.364811449	0.634936837	0.133179221
8	0.231632227		0.231632227
average Family size	5.549855188		

Rural 1981			
Parity i	F_i	p_i	B_i
0	1	0.997420088	0.002579912
1	0.997420088	0.984583953	0.015376275
2	0.982043813	0.95334088	0.045821301
3	0.936222512	0.902767078	0.09103165
4	0.845190862	0.8395189	0.135637159
5	0.709553703	0.772139173	0.161679494
6	0.547874209	0.706864043	0.16060163
7	0.387272578	0.646913248	0.136740817
8	0.250531762		0.250531762
average Family size	5.656109526		

Urban 1981			
Parity i	F_i	p_i	B_i
0	1	0.992326635	0.007673365
1	0.992326635	0.962341746	0.037369289
2	0.954957346	0.90471384	0.090994219
3	0.863963127	0.829027486	0.147713948
4	0.716249179	0.748911779	0.179841732
5	0.536407447	0.673446281	0.175165847
6	0.3612416	0.606423294	0.142176279
7	0.219065321	0.548472264	0.098914068
8	0.120151253		0.120151253
average Family size	4.764361908		

Table 7.5: The Parity Progression India, Bihar unadjusted

India 1981			
Parity i	F_i	p_i	B_i
0	1	0.977629228	0.022370772
1	0.977629228	0.913045835	0.085008933
2	0.892620295	0.819052991	0.161516973
3	0.731103322	0.720165181	0.204588166
4	0.526515157	0.630858191	0.194358757
5	0.332156399	0.555291857	0.147712656
6	0.184443744	0.492791967	0.093551348
7	0.090892395	0.441262192	0.050785018
8	0.040107378		0.040107378
average Family size	3.775467919		

Parity i	F_i	p_i	B_i
Urban India 1981			
0	1	0.977629228	0.022370772
1	0.977629228	0.913045835	0.085008933
2	0.892620295	0.819052991	0.161516973
3	0.731103322	0.720165181	0.204588166
4	0.526515157	0.630858191	0.194358757
5	0.332156399	0.555291857	0.147712656
6	0.184443744	0.492791967	0.093551348
7	0.090892395	0.441262192	0.050785018
8	0.040107378		0.040107378
average Family size	3.775467919		

Parity i	F_i	p_i	B_i
Rural India 1981			
0	1	0.990904723	0.009095277
1	0.990904723	0.956859826	0.042747802
2	0.948156921	0.894049884	0.100457336
3	0.847699585	0.81434088	0.157383159
4	0.690316426	0.732115295	0.184925212
5	0.505391214	0.656049226	0.173829699
6	0.331561515	0.589317241	0.136166598
7	0.195394917	0.532095586	0.091426144
8	0.103968773		0.103968773
average Family size	4.613394073		

Urban India 1971			
Parity i	F_i	p_i	B_i
0	1	0.988891003	0.011108997
1	0.988891003	0.949447933	0.049990484
2	0.938900519	0.880201802	0.11247859
3	0.826421929	0.795845344	0.168717885
4	0.657704044	0.711408768	0.189807621
5	0.467896424	0.634904543	0.170826858
6	0.297069565	0.568720061	0.128120144
7	0.168949421	0.512499368	0.08236295
8	0.086586472		0.086586472
average Family size	4.432419377		

Rural India 1971			
Parity i	F_i	p_i	B_i
0	1	0.996972445	0.003027555
1	0.996972445	0.982386858	0.017559818
2	0.979412628	0.948006112	0.050923471
3	0.928489157	0.893965328	0.098452044
4	0.830037113	0.828013156	0.142755463
5	0.68728165	0.759056077	0.165596337
6	0.521685313	0.693155135	0.160076459
7	0.361608853	0.633209255	0.132634781
8	0.228974073		0.228974073
average Family size	5.534461232		

Urban Bihar 1971			
Parity i	F_i	p_i	B_i
0	1	0.98772266	0.01227734
1	0.98772266	0.945308235	0.054020296
2	0.933702365	0.872716773	0.11884465
3	0.814857714	0.786090892	0.174305487
4	0.640552227	0.700670722	0.191736036
5	0.448816191	0.624060552	0.168727711
6	0.28008848	0.558233687	0.123733655
7	0.156354825	0.502570897	0.07777544
8	0.078579385		0.078579385
average Family size	4.340673847		

Rural Bihar 1971			
Parity i	F_i	p_i	B_i
0	1	0.990904723	0.009095277
1	0.990904723	0.956859826	0.042747802
2	0.948156921	0.894049884	0.100457336
3	0.847699585	0.81434088	0.157383159
4	0.690316426	0.732115295	0.184925212
5	0.505391214	0.656049226	0.173829699
6	0.331561515	0.589317241	0.136166598
7	0.195394917	0.532095586	0.091426144
8	0.103968773		0.103968773
average Family size	4.613394073		

Rural Bihar 1981			
Parity i	F_i	p_i	B_i
0	1	0.981684361	0.018315639
1	0.981684361	0.925370559	0.073262556
2	0.908421806	0.838703661	0.146525111
3	0.761896694	0.74357834	0.195366815
4	0.56652988	0.65515179	0.195366815
5	0.371163065	0.578908931	0.156293452
6	0.214869613	0.515075058	0.104195635
7	0.110673978	0.462020219	0.059540363
8	0.051133616		0.051133616
average Family size	3.966373013		

Urban Bihar 1981			
Parity i	F_i	p_i	B_i
0	1	0.975276474	0.024723526
1	0.975276474	0.906203984	0.091477048
2	0.883799426	0.80851703	0.169232539
3	0.714566887	0.707906797	0.208720131
4	0.505846756	0.618330811	0.193066121
5	0.312780635	0.543229619	0.14286893
6	0.169911705	0.481480651	0.088102507
7	0.081809198	0.430767337	0.046568468
8	0.03524073		0.03524073
average Family size	3.67923181		

Bihar 1981			
Parity i	F_i	p_i	B_i
0	1	0.981684361	0.018315639
1	0.981684361	0.925370559	0.073262556
2	0.908421806	0.838703661	0.146525111
3	0.761896694	0.74357834	0.195366815
4	0.56652988	0.65515179	0.195366815
5	0.371163065	0.578908931	0.156293452
6	0.214869613	0.515075058	0.104195635
7	0.110673978	0.462020219	0.059540363
8	0.051133616		0.051133616
average Family size	3.966373013		

Table 7.6: Mean age at birth Nepal and India

Nepal						
	1991	1986	1981	1977/78	1976	1971
A_1	21.1330	20.0113	20.0113	18.3734	20.2659	19.46335
A_2	25.1076	24.4412	26.3433	23.7965	23.7755	25.3012
A_3	28.3723	27.2555	29.5546	26.4461	26.4464	28.0674
A_4	31.1882	29.5571	32.0972	28.4205	28.4717	30.1821
A_5	33.6557	31.4563	34.1465	29.8592	29.9694	31.8174
A_6	35.9515	33.1464	35.9306	30.9837	31.1484	33.1921
A_7	38.3672	34.9288	37.7756	32.1156	32.3243	34.6196

India 1981					
	India Adjusted	Urban India Unadjusted	Rural India Unadjusted	Bihar Unadjusted	Urban Bihar Unadjusted
A_1	21.7192	20.6813	21.1204	22.3266	21.6570
A_2	25.7958	25.0591	25.8856	27.3965	26.4983
A_3	28.9776	28.5796	29.7926	31.2976	30.2791
A_4	31.6154	31.6910	33.1699	34.5292	33.5264
A_5	33.9481	34.8421	36.2101	37.5647	36.7947
A_6	36.3218	38.8876	39.27	41.1491	41.084
A_7	39.3341	45.8029	43.0136	46.8110	48.7046

	Rural Bihar Unadjusted
A_1	22.3697
A_2	27.4025
A_3	31.2364
A_4	34.3889
A_5	37.3439
A_6	40.8515
A_7	46.4419

Chapter 8

A Model of Parity

8.1 Introduction

Although Gompertz distribution is unimodal, a wide range of distributions including monotonically decreasing functions can be fitted by using truncated Gompertz distribution.
Here, a simple method of partial sums was employed to fit the Gompertz function.

Gompertz distribution

It is a continuous negatively skewed distribution over the entire real axis from $[-\infty, \infty]$. The distribution has two parameters B and c, and the probability density function has the form

$$g(u) = Ba^{c^u}c^u$$

where $-\infty < u < \infty$
The distribution function is

$$G(u) = 1 - a^{c^u}$$

where $-\infty < u < \infty$.
As seen from [50]

$$\mu_x = Bc^x$$

is called the Gompertz law.

$$\mu_x = -\frac{1}{l_x}\frac{dl_x}{dx}$$

$$= Bc^x$$

$\Rightarrow l_x = ka^{c^x}$ where $a = e^{-\frac{B}{lnc}} < 1$
k is a constant which ensures that $l_0 = 1$
Conditional probability of dying at age u given that a person survives till age x where $u > x$ is

$$\frac{l_u \mu_u}{l_x}$$

where

$$\mu_u = Bc^u$$
$$l_x = ka^{c^x}$$
$$l_u = ka^{c^u}$$

P(dying at age u |surviving till age x)= $a^{-c^x+c^u}Bc^u$

$$= Bc^u(a^{c^u-c^x})$$

where $u > x$,When x $\rightarrow -\infty$

$$g(u) = Bc^u a^{c^u}$$

where $-\infty < u < \infty$
This density function is called Gompertz distribution. The mirror image of the Gompertz density is $h(u)$

$$h(u) = g(-u) = Ba^{c^{-u}}c^{-u}$$

where $-\infty < u < \infty$

$$\gamma = c^{-1}$$
$$h(u) = Ba^{\gamma^u}\gamma^u$$

where $-\infty < u < \infty$
the corresponding distribution function

$$H(u) = a^{\gamma^u}$$

where $-\infty < u < \infty$

8.1.1 Application of Gompertz function to parity progression data

Parity progression ratios show maximal irregularity and sensitivity among and within population [37]. For less developed countries, they show a monotonical decrease, declining from a parity zero to a minimum at a highest parity. In developed countries on the other hand, they show a steep fall from zero to the parity two-three. Modeling and projecting parity progression is of considerable importance.

We recall the events from the previous chapter. p_i is the instantaneous probability of ith child. F_i is the probability of having atleast i children. It is a cummulative probabilistic function. $1 - F_i$ is the probability of having at the most i children. F_i is accumulated forwards whereas $1 - F_i$ is accumulated backwards. Here it is assumed that $1 - F_i$ followes Gompertz distribution. We apply the Gompertz distribution function to parity progression data. An additional factor A is taken into consideration as the sum of parity progression is not 1. The parity progression is denoted by F_i for given parity i. If $i = 1, \cdots 12$ then the data is divide into 3 partial sums over $i = 1, \cdots 4$ and $i = 5, \cdots 8$ and $i = 9, \cdots 12$

$$1 - F_i = A.a^{\gamma^i}$$

$$S_1 = \sum_{i=1}^{4}(1 - ln(F_i))$$

$$S_2 = \sum_{i=5}^{8}(1 - ln(F_i))$$

$$S_1 = \sum_{i=9}^{12}(1 - ln(F_i))$$

the estimates of γ , a and A are given in [50] .
The estimates are calculated in the following way.

$$ln(1 - F_i) = ln(A) + \gamma^i.ln(a)$$

$$S_1 - S_2 = ln(a)[\sum_{i=1}^{4}\gamma_i - \sum_{i=5}^{8}\gamma_i]$$

$$S_2 - S_3 = ln(a)[\sum_{i=5}^{8}\gamma_i - \sum_{i=9}^{12}\gamma_i]$$

thus,

$$\frac{S_1 - S_2}{S_2 - S_3} = \frac{1}{\gamma^n}$$

where $n = 4$, further

$$S_1 - S_2 = ln(a)[\sum_{i=1}^{4}\gamma_i - \sum_{i=5}^{8}\gamma_i]$$

$$ln(a) = \frac{S_1 - S_2}{(1 - \gamma^n)\sum_{i=1}^{4}\gamma^i}$$

Table 8.1: Parameters of Gompertz distribution fitted to parity progression Nepal

Year	A	a	$\hat{\gamma}$
1991	1.07539736	0.00017257	.66310998
1986	1.098519141	0.000119859	0.680345317
1981	1.1181741	9.07126E-05	0.692194149
1976	1.1181741	9.07126E-05	0.692194149
1977-78	1.111404727	9.95905E-05	0.688341364

$$\sum_{i=1}^{4}\gamma^i = \frac{\gamma(1-\gamma^n)}{1-\gamma}$$

$$ln(a) = \frac{(S_2 - S_1)(\gamma - 1)}{\gamma(1-\gamma^n)^2}$$

$$S_1 = n.ln(A) + ln(a).\sum_{i=1}^{4}\gamma^i$$

Thus ,

$$ln(A) = \frac{[S_1 - \sum_{i=1}^{4}\gamma_i.ln(a)]}{n}$$

$$\hat{A} = EXP[\frac{S_1S_3 - {S_2}^2}{n(S_1 - 2S_2 + S_3)}]$$

$$\hat{\gamma} = (\frac{S_3 - S_2}{S_2 - S_1})^{\frac{1}{n}}$$

$$\hat{a} = EXP[\frac{(\hat{\gamma}-1)(S_2 - S_1)}{\hat{\gamma}^{y+1}(\hat{\gamma}^n - 1)^2}]$$

$$\hat{A} = EXP[\frac{S_1S_3 - {S_2}^2}{n(S_1 - 2S_2 + S_3)}]$$

This simple estimate of partial sums, appear to provide estimates which are close to those estimated by maximum likelihood. The parameters calculated are then used to project F_i values. It should be noted that $F_0 = 1$ is not provided by this formula.

Table 8.2: R^2 and M.S.E, Nepal, Parity

Year	R^2	MSE
female 1991	0.991884782	0.001397376
female 1986	0.993387763	0.001052735
female 1981	0.993861146	0.000912269
female 1976	0.993861146	0.000912269
female 1977-78	0.993760523	0.000949693

Table 8.3: R^2 and M.S.E, India, Parity

Year	R^2	MSE
1981	0.995062352	0.000891429
1971	0.991177867	0.001264429

8.1.2 Interpretation of parameters

As seen from the diagram of A, a and c, the values lie roughly on a straight line. This linearity was used to extrapolate the values of A, a, c. For the calculation of the parameters and the observed and projected values, EXCEL worksheets were used intensively.

8.1.3 Results

The values of R^2 were close to 1, it could be interpreted that the fit is very good (Table 8.2-8.5). Very low value of M.S.E further verify this fact. Figures 63-72 reflect the fact that, the model has shown good results for Nepal, India, as well as Indian state Bihar.

8.1.4 Projection

The linearity of A, a, c was used to extrapolate the parity progression for Nepal. As seen from Table 8.1, the parameters A, a, $\hat{\gamma}$ lie roughly on a straight line when plotted against time. Using linear fit the values of A, a and $\hat{\gamma}$ are extrapolated, these extrapolated values are substituted in the equation. The projected values of Fi are given in Table 10.2.

Figure 63 - Comparison between observed and predicted Fis, Nepal, 1991, Gompertz model

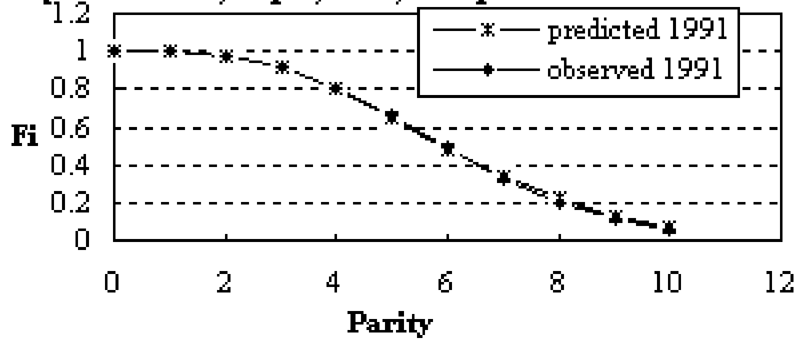


Figure 64 - Comparison between observed and predicted Fis, Nepal, 1986, Gompertz model

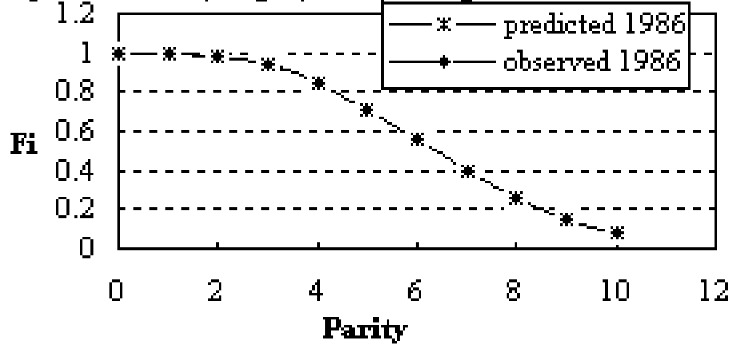


Figure 65 - Comparison between observed and predicted Fis, Nepal, 1981, Gompertz model

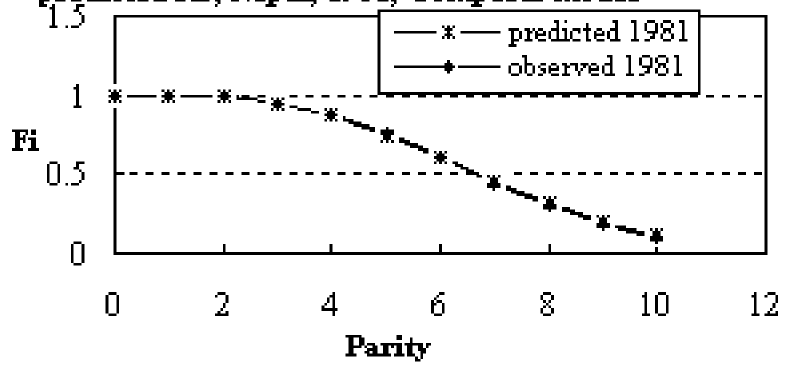


Figure 66 - Comparison between observed and predic Fis, Nepal, 1977/78, Gompertz model

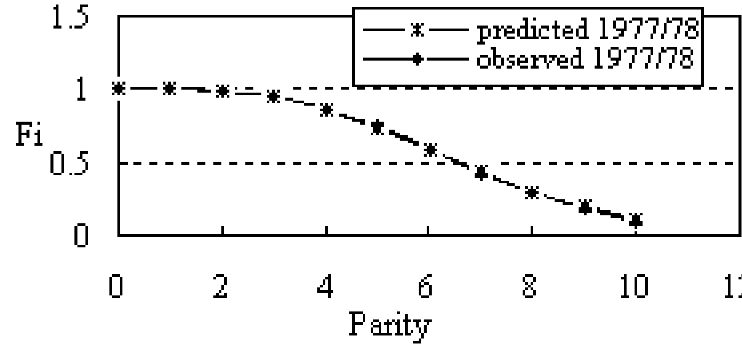


Figure 67 - Comparison between observed and predicted Fis, Nepal, 1976, Gompertz model

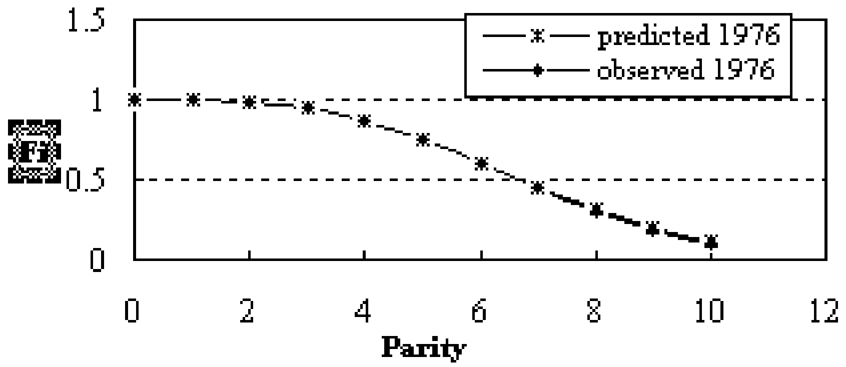


Figure 68 - Comparison between observed and predicted Fis, India, 1981, Gompertz model

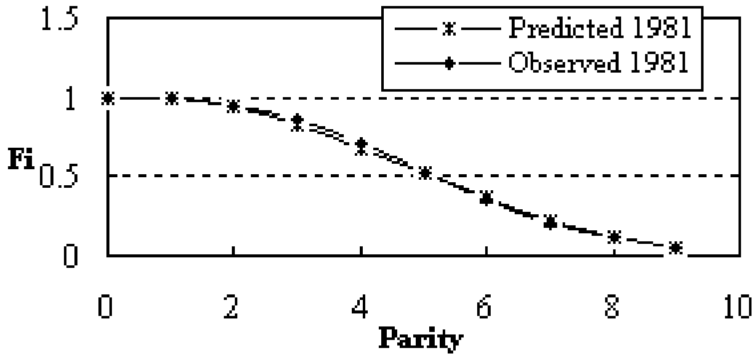


Figure 69 - Comparison between observed and predicted Fis, India, 1971, Gompertz model

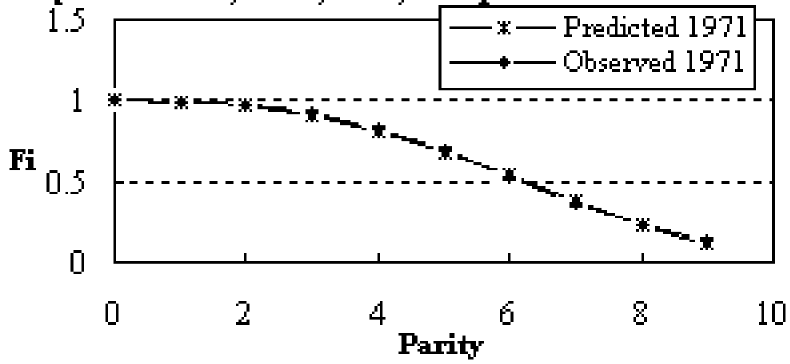


Figure 70 - Comparison between observed and predicted Fis, Bihar, 1981, Gompertz model

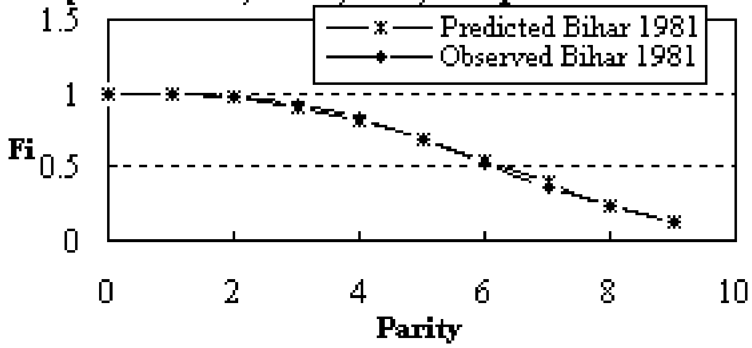


Figure 71 - Comparison between observed and predicted Fis, Rural Bihar, 1981, Gompertz model

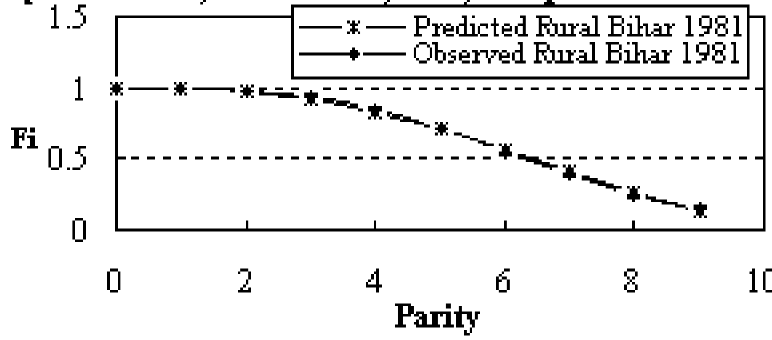


Figure 72 - Comparison between observed and predicted Fis, Urban Bihar, 1981, Gompertz model

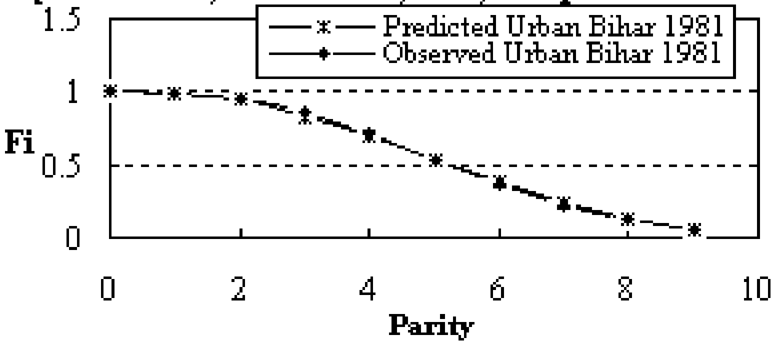


Table 8.4: Predicted F_i using Gompertz function Nepal

Parity(i)	F_i Predicted 1991	F_i Observed 1991	F_i Predicted 1986	F_i Observed 1986
0	1	1	1	1
1	0.9966	0.9963	0.9976	0.9975
2	0.9762	0.9756	0.9832	0.9826
3	0.9140	0.9176	0.9360	0.9380
4	0.7986	0.8094	0.8413	0.8488
5	0.6459	0.6578	0.7054	0.7149
6	0.4852	0.4881	0.5513	0.5543
7	0.3402	0.3297	0.4026	0.3937
8	0.2222	0.2030	0.2742	0.2560
9	0.1325	0.1143	0.1714	0.1528
10	0.06737	0.0591	0.0932	0.0839
11	0.0215	0.0282	0.0359	0.0426

Parity(i)	F_i Predicted 1981	F_i Observed 1981	F_i Predicted 1976	F_i Observed 1976	F_i Predicted 1977/78	F_i Observed 1977/78
0	1	1	1	1	1	1
1	0.9982	0.9982	0.9982	0.9963	0.9980	0.9980
2	0.9871	0.9866	0.9871	0.9756	0.9859	0.9854
3	0.9490	0.9502	0.94890	0.9176	0.94450	0.9464
4	0.8680	0.8736	0.8680	0.8094	0.8596	0.8658
5	0.7452	0.7531	0.7452	0.6578	0.7324	0.7408
6	0.5983	0.6012	0.5983	0.4881	0.5829	0.5859
7	0.4495	0.4418	0.4495	0.3297	0.4339	0.4258
8	0.3153	0.2982	0.3153	0.2030	0.3015	0.2840
9	0.2038	0.1852	0.2038	0.1143	0.1927	0.1741
10	0.1160	0.1061	0.1160	0.0591	0.1081	0.0984
11	0.0497	0.0563	0.0497	0.0282	0.0448	0.0514

Table 8.5: Predicted F_i using Gompertz function India

Parity(i)	F_i Predicted 1981	F_i Observed 1981	F_i Predicted 1971	F_i Observed 1971
0	1	1	1	1
1	.990	0.992	.995	0.997
2	0.944	0.952	0.976	0.979
3	0.819	0.857	0.901	0.928
4	0.676	0.706	0.805	0.83
5	0.521	0.524	0.684	0.687
6	0.366	0.349	0.541	0.522
7	0.227	0.209	0.387	0.362
8	0.118	0.113	0.239	0.229
9	0.049	0.056	0.119	0.133

(i) Par.	F_i Predicted Bihar 1981	F_i Observed Bihar 1981	F_i Predicted Rural Bihar 1981	F_i Observed Rural Bihar 1981	F_i Predicted UrbanBihar 1981	F_i Observed UrbanBihar 1981
0	1	1	1	1	1	1
1	.995	0.997	.995	0.997	.990	0.992
2	0.976	0.979	0.979	0.982	0.947	0.955
3	0.903	0.929	0.911	0.936	0.826	0.864
4	0.808	0.832	0.821	0.845	0.687	0.716
5	0.686	0.69	0.706	0.71	0.534	0.536
6	0.544	0.525	0.567	0.548	0.379	0.361
7	0.389	0.365	0.413	0.387	0.237	0.219
8	0.241	0.232	0.261	0.251	0.126	0.12
9	0.121	0.135	0.134	0.149	0.053	0.06

Chapter 9

Rural Urban effect, Fertility and Mortality

9.1 Rural-Urban effect, Fertility Nepal

9.1.1 Introduction

The rural and urban areas have shown a decline in fertility and mortality. The percentage decrease in urban areas is more than in rural areas. In rural areas, the women who exhibited decline in fertility are older.

Among the various socio-economic differentials of fertility studied, rural urban difference holds an important position.

While studying the rural and urban effect of fertility, it was assumed that women were exposed to rural-urban environment for a long time. This assumption was made because the effects of the environment on fertility may manifest after several years of exposure. For example, an urban woman, who has lived in the town for a long time is more likely to have developed attitudes favoring lower fertility and increased acceptance of contraceptive. On the other hand women who recently migrated from the countryside to the city may not have had time or opportunity to develop attitudes to favor smaller families. In Asia, the distribution of women across residence categories was extremely variable, with over 92 percent of women living in the rural areas in Bangladesh and Nepal [21].

9.1.2 Analysis

Here the total fertility rate is split into the contribution of rural areas and urban areas. The development of the formula can be found in [27].

If there are two factors α and β, the rate $F(\alpha,\beta)$ is a function of two factors. A special case is when when $F(\alpha,\beta)=\alpha.\beta$, $F(\alpha,\beta)=\alpha+\beta$

The factors take on values A, B and a, b in populations 1 and 2. In our case A, B denote the urban and rural Total Fertility rates for Nepal for the year 1991. a, b denotes the urban and rural Total Fertility rate respectively for Nepal for the year 1981.

$$F(a,b)-F(A,B)=TFR_{1981}-TFR_{1991}$$

$$F(a,b)-F(A,B)=\frac{[F(a,b)-F(A,b)]+[F(a,B)-F(A,B)]}{2}$$
$$+\frac{[F(a,b)-F(a,B)]+[F(A,b)-F(A,B)]}{2}$$
$$=\alpha effect+\beta effect$$

So, here a mathematical formula is constructed in which the mathematical contribution of each factor (rural and urban) is worked out. Total Fertility Rate (TFR) is the total number of children per woman in a population, assuming that women in reproductive age-span do not die.

$$TFR=\frac{B}{P}$$

Where B is the total number of births .
P is the population of women in reproductive age span.

$$B=B_r+B_u$$

where B_r is the total number of births in rural areas.
B_u is the total number of births in urban areas.

$$P=P_r+P_u$$

where P_r is the total population in rural areas.
P_u is the total population in urban areas.

$$TFR=\frac{B}{P}=\frac{B_r+B_u}{P_r+P_u} \tag{9.1}$$

Dividing the numerator and denominator of equation (9.1) by $P_r.P_u$

$$TFR=\frac{\frac{B_r}{P_r.P_u}+\frac{B_u}{P_r.P_u}}{\frac{P_r}{P_r.P_u}+\frac{P_u}{P_r.P_u}}$$

$$= \frac{\frac{TFR_r}{P_u} + \frac{TFR_u}{P_r}}{\frac{1}{P_u} + \frac{1}{P_r}}$$

Let $\frac{1}{P_u} = x_u$, $\frac{1}{P_r} = x_r$ $TFR_r = \alpha$, $TFR_u = \beta$

$$TFR = \frac{\alpha.x_u + \beta.y_r}{x_u + y_r} \tag{9.2}$$

So, the Total Fertility rate has been expressed as a weighted mean of Total Fertility rural and Total Fertility urban. The weights are x_u and y_r respectively. α effect =rural effect = $\frac{F(a,B)-F(A,B)+F(a,b)-F(A,b)}{2}$

So,we get:

$$F(a_{81}, B_{91}) = \frac{x_{r,91}.a_{81} + y_{u,81}.B_{91}}{x_{r,91} + y_{u,81}}$$

$$F(A_{91}, B_{91}) = \frac{x_{r,91}.A_{91} + y_{u,91}.B_{91}}{x_{r,91} + y_{u,91}}$$

$$F(a_{81}, b_{81}) = \frac{x_{r,81}.a_{81} + y_{u,81}.b_{81}}{x_{r,81} + y_{u,81}}$$

$$F(A_{91}, b_{81}) = \frac{x_{r,81}.A_{91} + y_{u,91}.b_{81}}{x_{r,81} + y_{u,91}}$$

β effect =urban effect = $\frac{F(a,B)-F(a,b)+F(A,B)-F(A,b)}{2}$ Substituting the calculated values of $F(a_{81}, B_{91})$, $F(A_{91}, B_{91})$, $F(a_{81}, b_{81})$, $F(A_{91}, b_{81})$ the β effect, or urban effect can be calculated. For simplification in calculation one more subscript (time factor)$x_{r,91}$ was considered.

9.1.3 Calculation

TFR Rural TFR	1981 1981 6.4	6.3 1991 5.8	1991 Urban	5.6 1981 5.8	 1991 3.5
Totalpopulation Women Rural	6885022	8456405	Urban	258642	813718

Source: Population Monograph of Nepal 1995.

TFR_r	b_{81}	6.4	B_{91}	5.8
TFR_u	a_{81}	5.8	A_{91}	3.5

Substituting the Fertility data in the equations above we get,

$$F(a, b) - F(A, B) = TFR_{1981} - TFR_{1991} = .7$$

$$F(a_{81}, B_{91}) = \frac{x_{r,91}.a_{81} + y_{u,81}.B_{91}}{x_{r,91} + y_{u,81}} = 5.8$$

$$F(A_{91}, B_{91}) = \frac{x_{r,91}.A_{91} + y_{u,91}.B_{91}}{x_{r,91} + y_{u,91}} = 5.598109324$$

$$F(a_{81}, b_{81}) = \frac{x_{r,81}.a_{81} + y_{u,81}.b_{81}}{x_{r,81} + y_{u,81}} = 6.378276526$$

$$F(A_{91}, b_{81}) = \frac{x_{r,81}.A_{91} + y_{u,91}.b_{81}}{x_{r,81} + y_{u,91}} = 6.093484622$$

α effect = rural effect = $\frac{F(a,B)-F(A,B)+F(a,b)-F(A,b)}{2} = 0.536825$
β effect = urban effect = $\frac{F(a,B)-F(a,b)+F(A,B)-F(A,b)}{2} = 0.243341$

9.1.4 Interpretation

Various social and economic factors contribute in varying extent to the vital rates. In many developing countries including Nepal, the majority of the population lives in rural areas. In a high fertility country, people living in the villages make a major contribution to the fertility pattern. The reasons are, lack of contraceptive awareness, lack of medical facilities, and the percentage of people inhabiting there. As seen from equation, the rural areas contribution to the total country's fertility is 68.80908 percent and urban areas contribution is 31.19091514 percent.

9.2 Rural-Urban effect in Mortality India

9.2.1 Introduction

As mentioned in [1] the 967 million population of India will surpass the population of China by the middle of the 21st century. It's growth rate is 1.7 percent, which is about 70 percent more than of China. In addition to that India has more adults living with HIV/AIDS, estimated 2-5 million in mid-1996.

Like many other developing countries, India has achieved substantial improvement in mortality, thanks to the advancement in the medical facilities. The life expectancy has risen from 32-33 years in 1947 to 62 years in 1997. The infant mortality rate has dropped down from 200-225 per 1000 to 66 per thousand in the same period.

India's states vary widely with respect to fertility, mortality and contraceptive use. In addition to rural urban differential, there is a north-south gradient. Here the rural urban effect with respect to mortality is studied.

According to [20], infant mortality and fertility are positively correlated, that is infant mortality is generally high where fertility is high. Since fertility tends to be lower in urban areas than in rural areas, infant mortality would consequently be lower in urban areas. Urban areas have good water supply as compared to the countryside. Good transportation and communications also contribute in lowering the mortality. In addition, the public health measures are well developed and implemented. In India, there was one physician to every 1400 people in urban areas as opposed to one for every 18000 in the rural areas in 1960. By 1971 there was one physician for every 1500 people in urban areas compared to one for every 12000 in rural areas. In 1973-74, although the number of hospital beds had more than doubled since 1950, there were only two beds for every 10,000 people in rural areas compared to 17 beds in urban areas.

These facts exhibit the seriousness of rural urban differential in less developed countries, in such an important sector as health.

Urban areas have their own problems. They are over filled with population growing rapidly and with people who have migrated from the rural areas to the urban areas. There are inadequate sewage disposal facilities, poor pure drinking water facilities and housing facilities.

9.2.2 Analysis

Here the ${}_nq_x$, the conditional probability of dying in the age group (x, x+n], given that a person survives till age x, is split into the contributions of rural areas and urban areas to this probability.

Let us assume that l_0 people are born at time t irrespective of rural or urban area. So, the cohort is l_0.

$$ {}_nq_x = \frac{{}_nd_x}{lx}|_{cohortl_0} $$

where ${}_nd_x$ is number of people out of the cohort of l_0, dying in age group. $(x, x+n]$.
l_x is the number of people out of the cohort of l_0, living till age x.

$$ \begin{aligned} {}_nq_x &= \frac{{}_nd_{xr} + {}_nd_{xu}}{lx_r + l_{xu}}|_{cohortl_0} \\ &= \frac{({}_nq_x)_r(\frac{1}{l_x})_u + ({}_nq_x)_u(\frac{1}{l_x})_r}{(\frac{1}{l_x})_u + (\frac{1}{l_x})_r} \end{aligned} $$

It further simplifies to the weighted average of $({}_nq_x)_r$ and $({}_nq_x)_u$, the weights being $({}_nl_x)_r$ and $({}_nl_x)_u$ respectively.

9.2.3 Application of the method

The percentage of people living in rural areas and the percentage of people living in urban areas is assumed to be constant over all ages. Let l_0 be the total number of births in the whole country, irrespective of the rural and urban areas. Let r_1 be the percentage of the total births in rural areas, r_2, the percent of total birth in urban areas. The $\frac{l_0 r_1}{100}$ is the cohort for rural areas, denoted by $(l_0)_r$, and $\frac{l_0 r_2}{100}$ is the cohort for urban areas, denoted by $(l_0)_u$. From the data source, SRS based Abridged Life Table, we have abridged life table rural and urban areas 1988-92 and 1991-95 when the cohort is $l_0 = 100,000$. Then the new life table when the cohort is $(l_0)_r$ and $(l_0)_u$ can be easily constructed. Values from these new life tables can be substituted in equation above to find the rural urban effect. In 1988 the percent population in rural areas 74 and percent population in urban areas 26(source Census India). Taking the assumption into consideration if l_0(whole country)$= 100,000$ then in 1988 l_0(rural)$= 75000$ and l_0(urban)$= 25000$. In 1991 the percent population in rural areas was 75 and percent population in urban areas was 25(source Census India)then in 1992 l_0(rural)$= 74000$ and l_0(urban)$= 26000$

On the basis of these Tables (9.1-9.2) the rural and urban effects are calculated . The calculated age specific rural and urban effects on nqx are given below.

$$F(a,b) - F(A,B) =_n q_x 1981 -_n q_x 1991$$

Then using the formula used to calculate rural and urban effect for ASFR(age specific fertility rate), the rural and urban effects for ${}_nq_x$ can be calculated(for given n and x)(Table 9.3).

9.2.4 Interpretation

The rural and urban effect on the mortality $({}_nq_x)$ showed that the rural area overwhelmingly affected the country's mortality(Figure 73). The effect of rural areas and urban areas on the mortality was studied here for each age-group individually. For the age groups 0-1, 5-10, 10-15 15-20, 20-25, 25-30, 30-35, 35-40, 40-45, 65-70 the rural areas explained 91, 79, 89, 75, 90, 70, 74, 64, 76, 50, and 52 percent respectively of the country's total mortality.As the urban areas, compared to the rural areas are far ahead in medical facilities, the number of people surviving at older ages are also higher in urban areas.

Table 9.1: Derived life Table, 1991

Age-interval	l_x(rural)1991	d_x(rural)1991	${}_nq_x$(rural)
0-1	74000	6390.64	0.08636
1-5	67609.36	2720.98	0.040245611
5-10	64888.38	898.36	0.013844698
10-15	63990.02	481.74	0.007528361
15-20	63508.28	682.28	0.010743166
20-25	62826	892.44	0.014204947
25-30	61933.56	955.34	0.01542524
30-35	60978.22	1040.44	0.017062486
35-40	59937.78	1154.4	0.019259973
40-45	58783.38	1475.56	0.025101653
45-50	57307.82	2109	0.03680126
50-55	55198.82	3098.38	0.056131272
55-60	52100.44	4326.04	0.083032696
60-65	47774.4	6262.62	0.131087361
65-70	41511.78	7790.72	0.18767492
70+	33721.06	33721.06	

Age-interval	l_x(urban)1991	d_x(urban)1991	${}_nq_x$(urban)
0-1	26000	1385.28	0.05328
1-5	24614.72	487.24	0.019794659
5-10	24127.48	151.58	0.006282463
10-15	23975.9	111.28	0.004641327
15-20	23864.62	153.4	0.006427926
20-25	23711.22	217.1	0.009156003
25-30	23494.12	232.7	0.009904606
30-35	23261.42	273	0.011736171
35-40	22988.42	320.84	0.013956592
40-45	22667.58	442.26	0.019510684
45-50	22225.32	641.42	0.028859877
50-55	21583.9	999.96	0.046328977
55-60	20583.94	1443.26	0.070115828
60-65	19140.68	2156.18	0.11264908
65-70	16984.5	2663.44	0.15681592
70+	14321.06	14321.06	

Table 9.2: Derived life tables, 1988

Age-interval	l_x(rural)1988	d_x(rural)1988	${}_nq_x$(rural)
0-1	75000	6816	0.09088
1-5	68184	3423.75	0.050213393
5-10	64760.25	1028.25	0.015877795
10-15	63732	523.5	0.008214084
15-20	63208.5	735	0.011628183
20-25	62473.5	831.75	0.013313645
25-30	61641.75	921	0.014941172
30-35	60720.75	1002.75	0.016514124
35-40	59718	1197.75	0.020056767
40-45	58520.25	1557	0.026606175
45-50	56963.25	2187	0.038393175
50-55	54776.25	3118.5	0.056931608
55-60	51657.75	4410.75	0.08538409
60-65	47247	6339.75	0.134183123
65-70	40907.25	8130.75	0.198760611
70+	32776.5	32776.5	

Age-interval	l_x(urban)1988	d_x(urban)1988	${}_nq_x$(urban)
0-1	25000	1341	0.05364
1-5	23659	557.5	0.023563971
5-10	23101.5	172.75	0.007477869
10-15	22928.75	98.5	0.004295917
15-20	22830.25	161.5	0.007073948
20-25	22668.75	197.5	0.008712435
25-30	22471.25	231.5	0.010302053
30-35	22239.75	245.5	0.011038793
35-40	21994.25	329.75	0.014992555
40-45	21664.5	445.25	0.020552055
45-50	21219.25	693.75	0.032694369
50-55	20525.5	1063.5	0.051813598
55-60	19462	1630.25	0.0837658
60-65	17831.75	2176.75	0.122071586
65-70	15655	2826	0.180517407
70+	12829	12829	

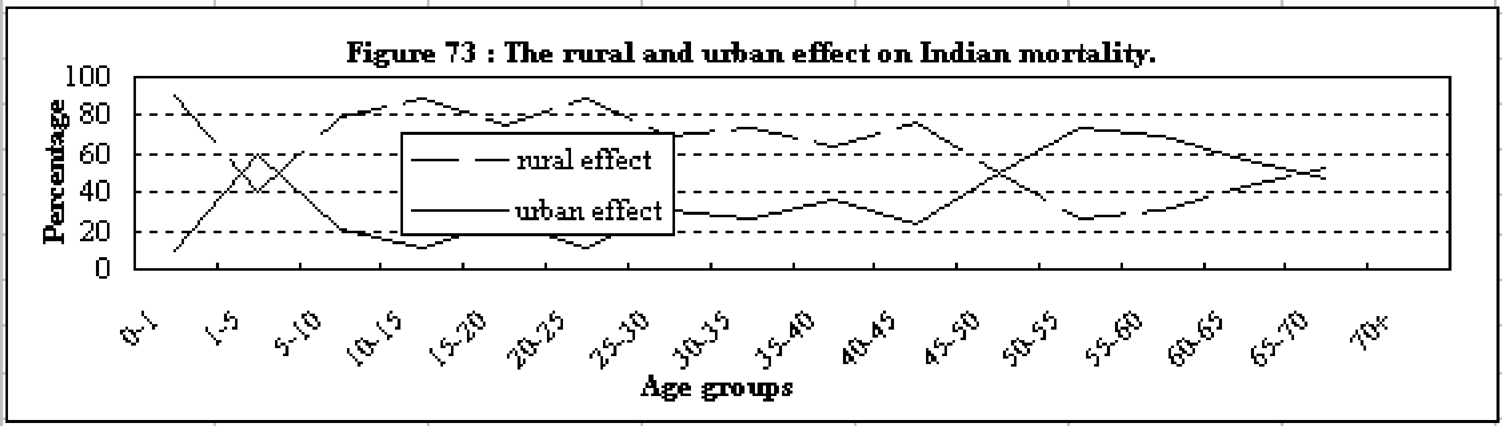


Table 9.3: Percentage rural and urban effects

Age-interval	rural effects	urban effects
0-1	90.71872496	9.281275036
1-5	40.95697364	59.04302636
5-10	79.34962494	20.65037506
10-15	88.80016436	11.19983564
15-20	75.1399017	24.8600983
20-25	89.48663737	10.51336263
25-30	70.11375567	29.88624433
30-35	74.01460684	25.98539316
35-40	63.65294695	36.34705305
40-45	76.27348041	23.72651959
45-50	50.57442245	49.42557755
50-55	26.25205036	73.74794964
55-60	30.25359001	69.74640999
60-65	43.44355908	56.55644092
65-70	52.4668218	47.5331782

In other words the people living in rural areas have a much lower life expectancy than those living in urban areas. This perhaps explains the high contribution of urban areas to the country's overall mortality at older ages. At age-groups 50-55, 55-60, 60-65, the urban mortality explains 74, 70, 57 percent respectively of the total mortality. Infant mortality is far higher in the countryside than in the cities. So the number of infants surviving to the age group 1-5 is higher in cities than in villages. This is perhaps responsible for the high contribution of urban areas to the countries total mortality (59 percent) as compared to the rural contribution of (41 percent).

Chapter 10

Projection

10.1 Projection

W.Brass [19] defined prediction as a procedure in which a well established functional model over time is fitted by the use of past and current measures for a particular population and the implications extrapolated forward. The past and current measures may be variables intrinsic to the process or extraneous influencing factors.

In the countries with abundant and accurate data, the past trend of the parameters can be studied and it can be further extrapolated into the future path. But the number of countries with sufficient and accurate records are very few. In the case of countries with limited data, the trend of the parameters are only a short term guide to the future change. Thus, it can be concluded that the past for a particular population provides only rather broad information about it's future and, consequently, a very limited range of specification of paths of change is justified.

According to Nathan Keyfitz, page 107-121 [19], Projections are broadly of two types, long term projection and short term projection. While projecting, even if the available data is few and faulty, the intractability of the real world has to be considered. For the success of a demographic forecast, it should be combined with an economic forecast. This will prevent the failing of the model at the turning points, resulting from economic, technical and social changes. In other words, the intuitive knowledge refined by ecological and other theory should be incorporated in the long-term projection.

Short-term projections:

These are mainly extrapolation of population numbers based on recent past. It also involves the extrapolation of the components separately, inclusion of age, projection of death by cause, transformation of deaths into probabilities and then into Logit and other functions.

Long-term projections:

These are derived by interpolating between the present and some assumed ultimate condition. One can through long term projections, by taking the economic, environmental and material problems of a country make a long-term projection of the population.

Projection of Mortality in the United Nations:

As written in [19], page 12-41, while projecting the mortality for developing countries, the mortality projection was based on either the United Nations model life tables or Coale - Demeny regional model life tables. But the use of modified series of model progressions of mortality improvement, which was considerably slower than the scheme of time paths set out in the United Nations model life tables was made. The reason for this use was the fact that mortality improvement registered in those countries with reliable death statistics have suggested that the rapid increase in life expectancy which occurred during the 1950s was not repeated during the 1960s. That finding was anticipated in the countries, where the level of mortality was already low in 1960s. There was evidence suggesting that the countries with moderately high mortality level at the outset of 1960s did not experience a decline during the decade as rapid as those anticipated in the 1950s by other developing countries, where the transition to low mortality level had begun earlier. Thus a modification was necessary. According to the working model , in the United Nations model life tables a quinquennial gain of 2.5 years in life expectancy at birth Was assumed until the life expectancy had reaches 55 years. It was further assumed that the highest male life expectancy was 72.6 years, the highest female life expectancy was 77.6 years.

United Nations methodology for projection and estimation:[22]

Estimation Procedure:

A lot of efforts have to be taken by the Population Division, for the evaluation and adjustment of the demographic data collected from many countries specially from the less developed region. These data are often incomplete or deficient or internally inconsistent. The adjustment of the data is the step which further leads to it's further preparation for projections and publications. For such adjustments, the Population division of the United Nations relies upon external sources such as post-enumeration surveys and special studies on the completeness of the vital registration.

An important step in the estimation process is the determination of the size of the total population and it's age and sex structure to the date closest to the base year, that is, 1995. Here the past estimates of fertility, mortality and migration have to considered simultaneously. The reason behind it is the interrelation between all these variables. Past estimates of the total population (1950-1995) and it's age and sex structure are also examined and adjusted (taking the base year into consideration) . Further the information from previous censuses, civil registrations and surveys, or the age and sex structures of similar countries, including those derived from stable population models, are taken into account.

Projection Procedure:

Population projections are carried out at five year intervals from 1995 to 2050 according to the three fertility variants and the constant fertility scenario. In this process, assumptions regarding future trends in fertility, mortality and international migration have to be made. National population projections' assumption are considered, but some adjustments have to be made, suiting to the United Nations needs of international compatibility. The extent and accuracy of the initial population data for the base year are important for short term projections.

The three fertility variants are referred to as high, medium and low, depending upon the assumed levels of future fertility. The low- and high fertility variants are usually thought to bracket the future population change of each country. The fertility pattern could frequently go outside this high and low fertility range. The medium fertility variant implies medium pattern of fertility change.

Cohort component method is used to prepare projections by sex and age for all countries with a population of 150,000 people or more in 1995. For countries with population less than 150,000 in 1995, projections are made for total population only. Cohort component technique considers fertility, mortality and international migration-together with the base year population by age and sex.

This method comprises of the following major steps:

1. The number of survivors in each age and sex category at the end of each five year period is calculated by applying age and sex specific survival rates to the base year population. The survival rates are usually derived from an observed or estimated national life table, adjusted to match future life expectancies at birth through assumption of age - sex pattern of mortality change. In cases where no national life table is available, survival ratios from the model life tables must be accepted.

2. The number of births expected to take place during each five - year

period is calculated by applying projected age- specific fertility rates to the average number of women in each five -year age groups. As with mortality, the age specific fertility rates are usually derived observed and estimated national fertility pattern, adjusted to match projected total fertility rates through assumptions of age specific fertility change.

3. The assumed number of net international migration is also incorporated into the calculation.

Mortality Assumptions:

Three working models of mortality have been developed according to the pattern of change in the life expectancy. They are a fast life expectancy rise, a middle life expectancy rise, a slow life expectancy rise. One of these models usually describes the mortality pattern of most of the countries. Under special situations sometimes the projected course departs from these models. After 2025, life expectancy at birth in all countries is assumed to rise according to the middle mortality schedule. All these models assume that the improvement becomes slower as the life expectancy increases. The highest life expectancy at birth allowed in these models is 87.5 years for females and 82.5 for males. The middle model, which is most often used, assumes that male life expectancy at birth will increase by 2.5 years every quinquennium until it reaches 60 years and that the quinquennial gain is then gradually reduced to .4 years at life expectancy at birth of 77.5 years and remain at 0.4 thereafter. For females, the life expectancy at birth is assumed to increase by 2.5 years every quinquennium until it reaches 65 years, and then it reduces to 0.4 year at a life expectancy of 82.5 years.

Fertility Assumptions:

For countries exhibiting fertility rates above replacement level, the United Nations generally assumes that TFR will begin to decline or continue to decline, but the decline will pause at some time in the future. Fertility is assumed to reach and pause at the replacement level of 2.1 children per women , in the medium variant. It is assumed to pause at 2.6 in the high variant and at 1.6 in the low variant.

For countries with below replacement level a different procedure is used. For such countries A target period is not determined a priori; instead a future pace of fertility is assumed. The pace and direction of future fertility change depends upon the recent fertility experience of the country. TFR was assumed to continue declining at it's past pace through 1995-2000 and then remain at it's 1995-2000 level through 2000-2005. As a result, TFR was

Table 10.1: Distribution of population, Nepal 1971-1991

Age	1971	1981	1991
00-04	791139.608	1192777.08	1373925.126
05-09	884214.856	1115823.72	1429250.97
10-14	703881.563	915744.984	1207947.594
15-19	546817.082	692580.24	875992.53
20-24	465376.24	638712.888	728456.946
25-29	453741.834	569454.864	645468.18
30-34	383935.398	469415.496	553258.44
35-39	383935.398	461720.16	516374.544
40-44	302494.556	377071.464	414943.83
45-49	244322.526	330899.448	378059.934
50-54	203602.105	292422.768	304292.142
55-59	133795.669	184688.064	248966.298
60-64	139612.872	192383.4	212082.402
65+	174516.09	261641.424	331955.064

assumed to rise towards it's replacement level at a pace of .07 child per 5 years. If recent fertility was relatively constant, TFR was assumed to remain constant through 1995-2005 then to increase towards replacement level at a pace of 0.7 child per projection period. If fertility was rising recently, TFR was assumed to continue rising at a pace of .07 child per projection period, until it reaches 2.1.

10.2 A note on the defective data:

While analysing the data for modeling and projection, various instances of inaccuracies and errors were observed. For example, the age specific population data (Table 10.1) reflect over-estimation and under-estimation. The successive age groups in successive time period show a big fluctuation. As seen in the table, the people in the age group 0-4 in 1971 will be in the age group 10-14 in 1981. On the account of mortality (or any other such decrements). The amount in this age group 10-14, should be smaller than the amount in age group 0-4. In such an age group normally massive immigration is also not possible. But the data show inconsistancies at various stages.

Table 10.2: Projected ASFR 2001-2015; Nepal

Age	**2001**	**2005**	**2011**	**2015**
17	0.0431	0.0376	0.0294	0.0246
22	0.0974	0.0914	0.07845	0.0695
27	0.1824	0.1767	0.1616	0.1499
32	0.2569	0.2528	0.2440	0.2362
37	0.2288	0.2405	0.2521	0.2572
42	0.0906	0.1290	0.1629	0.1806
47	0.0070	0.0298	0.0583	0.0750

Table 10.3: Projected F_i, Parity-progression, Nepal, 2000-2015

Age	**2000**	**2005**	**2011**	**2015**
0	1	1	1	1
1	0.996	0.995	0.994	0.993
2	0.970	0.966	0.959	0.956
3	0.896	0.883	0.866	0.854
4	0.766	0.743	0.714	0.696
5	0.604	0.574	0.541	0.520
6	0.441	0.413	0.382	0.363
7	0.302	0.278	0.255	0.241
8	0.194	0.177	0.162	0.154
9	0.114	0.104	0.098	0.095
10	0.059	0.055	0.055	0.057
11	0.021	0.022	0.027	0.032
12	0.0089	0.00075	0.0096	0.017
family-size	5.36	5.21	5.06	4.98

Table 10.4: Projected Cumulative-ASFR 2001-2015; Nepal

Age	**2001**	**2005**	**2011**	**2015**
17	0.1186	0.1244	0.1334	0.1396
22	0.3792	0.3857	0.3946	0.3999
27	0.6626	0.6612	0.6572	0.6534
32	0.8662	0.8547	0.8355	0.8216
37	0.9852	0.9658	0.9353	0.9142
42	1.0480	1.0237	0.9864	0.9609
47	1.0795	1.0525	1.0113	0.9836

Chapter 11

Summary

The objective behind this study was to mathematically analyse, model and forecast the vital rates (mortality and fertility) of Nepal. In order to attain this goal, the data have been converted into tables and analysed intensively using several softwares such as Mocrosoft Excel, SPSS, Mathematica. The margin of error of data has been analysed. In Chapter 4, the error and uncertainity in the data have been analysed using Bayesian analysis. The reliability of the data of Nepal has been compared with the reliability of the data of Germany. The mortality and fertility conditions of Nepal have been compared from two angles. Data on India (particularly north India) have provided comparison on the socio-economic grounds whereas data on Germany(with accurate and abundant data) have provided comparison on the ground of data availability and accuracy. Thus in addition to analysing and modeling the data, the regional behaviour has been studied. The limited and defective data of Nepal have posed a challange at every stage and phase. Because of this very long term forecasting of mortality could not be made. But the model has provided a lot of information on the mortality for the years for which the data were lacking. But in the comming future, with new data at hand and with the new models developed here, it could be possible to do long term projections. In the less developed world, rural and urban areas have a big impact on the mortality and fertility of a country. The rural and urban effects on mortality and fertility have been studied individually.

While analyzing the mortality scene of Nepal, it has been observed that the mortality is decreasing. The decrease is slow, but it reflects the advancement in medical facilities and health awareness. The fertility is also decreasing. There is a decrease in the number of children per woman and per family. This decrease is more pronounced in the urban areas as compared to the rural areas. This also reflects that the family planning programmes launched are showing results, particularly in urban areas.

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